Dynamical analysis of an intermittency in an open cavity flow

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When open flows pass an open cavity, it is known that for medium or large Reynolds numbers, self-sustained oscillations generally appear. When more than one mode is excited, some nonlinear competition between modes may occur. In the configuration investigated here, the underlying dynamics are mainly driven by two dominant modes. The interplay between these two modes is investigated using phase portraits, Poincaré sections, and return maps. The toroidal structure of the phase portrait is then investigated using a symbolic dynamics built from an angular return map. Each symbol can be associated with a specific mode and the interplay described in terms of symbolic sequences, leading to exhibit a switching mode process. © 2008 American Institute of Physics. [DOI: 10.1063/1.3005435]

I. INTRODUCTION

Open flows over a cavity present a dynamics resulting from a complex feedback process leading to sustained oscillations. This is typical of unstable configurations in which a preferred mode is selected and amplified.¹ Usually, these oscillations are not welcome. It is therefore important not only to understand their origin but also to characterize the underlying dynamics for the possibly of being able to control the flow in order to reduce these instabilities. Indeed, previous works showed that these oscillations are related to a mode-switching phenomenon² which remains to characterize from a dynamical point of view. Such a dynamical characterization could also provide useful information to validate some assumptions made in some numerical simulations as those carried out at LIMSI.³

In this paper, the dynamics underlying the flow of an open cavity at medium Reynolds number is investigated using some tools borrowed to the nonlinear dynamical system theory. Thus, phase portraits, Poincaré sections, and various return maps will be used as well as a symbolic dynamics based approach. This latter technique was already used to distinguish two regimes in a free water jet.⁴ It will be shown that the approach using a symbolic dynamics provides clear results without the ambiguity left by more “classical” techniques.⁵

The subsequent part of this paper is organized as follows. Section II is devoted to the experimental setup. In Sec. III a brief frequency analysis is discussed and the dynamical analysis of the flow is developed in Sec. IV. Section V gives a conclusion.

II. EXPERIMENTAL SETUP

The system under study consists in a cavity of length \( L = 10 \text{ cm} \) orientated along the \( x \)-axis, that is, the direction of the incoming air flow. The height of the cavity—oriented along the \( y \)-axis—is such as the aspect ratio \( R = (L/H) \) is equal to 2. The width of the cavity—\( W = 30 \text{ cm} \)—is along the \( z \)-axis [Fig. 1]. The cavity is embedded within a vein of total height \( D + H = 12.5 \text{ cm} \). The origin of the plane \( Oxy \) is located at upstream edge of the cavity, and at the middle of the cavity along the \( z \)-axis. The air flow is produced by a centrifugal fan located at the entry of the settling chamber [Fig. 1(a)]. The incoming boundary layer is laminar and stationary. The external velocity \( U_e \) is measured using a laser Doppler velocimeter (LDV) whose probe is located at 102 mm upstream the cavity and 25 mm above the bottom of the vein. Locating the probe sufficiently upstream of the cavity avoids in the velocity measurement any perturbations induced by the instabilities caused by the cavity. The air flow leaves the wind tunnel directly in the atmosphere. The reference velocity is \( U_e = 2.09 \text{ m s}^{-1} \), that is, the flow has a Reynolds number \( Re_U = (U_eL/v) \approx 14 \text{ 000} \).

The probe used for measuring the \( x \)-component of the velocity within the cavity is located at \((x = 1.15L, y = 0.33H, \text{ and } z = 0)\). The seeding rate of the flow leads to a mean time of flight between two particles of 0.654 ms. The time series is then resampled using a linear interpolation between the measured data points in such a way that successive data points are equally distant in time by \( dt = 1/f_s = 0.654 \text{ ms} \), with the sampling rate \( f_s = 1530 \text{ Hz} \). The acquisition time is about 9 min giving a time series length of about 840 000 data points.

III. FREQUENCY ANALYSIS

The development of the flow dynamics is driven by the main control parameter constituted by the external velocity \( U_e \). An overview of that evolution is given by the power spectral density evolution with increasing \( U_e \) from 0.6 to 2.6 m s\(^{-1}\) (Fig. 2). At low external velocity, a first spectral mode (say \( f_0 \)) increases and then disappears, while a second spectral mode (say \( f_1 \)) appears to reach a maximum amplitude at \( U_e = 1.7 \text{ m s}^{-1} \). Around that external velocity, a third frequency mode (say \( f_2 \)) just starts to grow. So doing, only main frequencies produced by the flow instabilities are considered. However the power spectral density, in fact, contains other low frequencies and harmonics that will not be explic-
The first step when investigating a dynamical system is to reconstruct the phase space from the time series measured by the LDV technique. There are different coordinate sets which can be used to reconstruct the phase space, delay, or derivative coordinates as mentioned in the earlier works or principal components based on the singular value decomposition of a delayed data matrix. Gibson et al. showed that all these coordinate sets are equivalent. We choose to use principal components (the $X_i$ in what follows). The dimensionality of the phase portrait can be estimated using a Grassberger–Proccacia algorithm. A correlation dimension is thus found to be about $4.2$ ($U_e=2.09 \text{ m s}^{-1}$), which means that, to be ensured to have a diffeomorphism between the original—unknown—phase space and the reconstructed phase space, a 10D space should be used according to the Takens’ criterion. This is a too large dimension to be able to figure out the structure of the underlying dynamics. What this correlation dimension tells us is that the structure cannot be interpreted in terms of branched manifolds—templates—as described by Gilmore and Lefranc. However, as it will be shown below, it is possible to gain more insights on dynamical processes driving this complex dynamics from a projection of the reconstructed phase space spanned by the first two principal components.

The phase portrait (Fig. 4) presents a toroidal structure, that is, a trajectory which is mainly organized around a torus. A first-return map to a Poincaré section appears as a cloud of points (Fig. 5). This means that the torus—if it exists for other parameter values—is already destabilized. The trajectory therefore visits all the space delimited by such a torus and a cloud of points is observed in a first-return map to a Poincaré section rather than an annular structure. The phase portrait has therefore some reminiscences of the toroidal structure but is organized around a much more complicated set of manifolds. From the return map it is quite difficult to distinguish a deterministic—eventually contaminated by a noisy component—dynamics from a stochastic process.

It was shown in the dynamical analysis of a water jet experiments that a toroidal structure can be conveniently investigated using an angular first-return map. Such a map is built on the angle $\theta_n$ associated with the $n$th iterate of the first-return map as follows. At each point of the first-return map (Fig. 5) an angle $\theta_n$ is defined as the angle between the segment joining that point to the barycenter of the first-return map, and the right-hand half-line from the barycenter. Then $\theta_{n+1}$ is plotted versus $\theta_n$. A probability density function $P(\theta_n)$ is also computed (Fig. 6). It clearly appears that the dynamics is mainly organized around two main neighborhoods in the Poincaré section corresponding to the two peaks of the probability density function $P(\theta_n)$. These two neighborhoods are located along the bisecting line, that is, they surround two period-1 points. Indeed, points close to the bisecting line

**IV. DYNAMICAL ANALYSIS**

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are such that $\theta_{n+1} = \theta_n$. This means that the trajectory is in the neighborhood of a period-1 orbit. Since period-1 orbits are involved, it is possible to associate with them a characteristic frequency of oscillations. Consequently, points located in the center of each cloud and in the first bisecting line should correspond to a realization of one of the modes identified with the spectral analysis. Other points—far from the bisecting line—are associated with transitions from one mode to the other. Rigorously, only points in the bisecting line and at the top of the peak in the probability visits (Fig. 6) should be taken into account to estimate the mean time duration of oscillations associated with each of the two

![Phase portrait](image1)

**FIG. 4.** Phase portrait spanned by the first two principal components reconstructed from the measurements of the horizontal component of the velocity.

![First-return map](image2)

**FIG. 5.** First-return map to a Poincaré section of the phase portrait spanned by the first two principal components reconstructed from the measurements of the x-component of the velocity. The Poincaré section is defined by $X_1 = 0$ and $X_1 > 0$. 
modes, $f_1$ and $f_2$. The mean duration between two successive crossings of the Poincaré section, lying in the angular sector $\theta_n \in [-(\pi/4), 3\pi/4]$ is 0.0335 s, corresponding to 29.9 Hz, rather close to $f_2=31.0$ Hz. Similarly, events associated with the angular sector $\theta_n \in [-(\pi/2); -(\pi/4)] \cup [3\pi/4; 3\pi/2]$ have a mean duration of 0.0423 s, corresponding to 23.6 Hz, very close to $f_1=23.2$ Hz. Henceforth, it is reasonable to consider that the events at frequency $f_2$ essentially contribute to the first partition, while events associated with the second partition are essentially due to frequency $f_1$. These two main frequencies differ by less than 3% of the values obtained with a simple Fourier analysis of the time series. Each partition therefore seems to be associated with one of each mode evidenced with the Fourier spectrum [Fig. 3(a)].

In order to investigate in a deeper way how the dynamics is switching from one partition to the other, a symbolic dynamics is now built on the partition of the angular first-return map. The evolution of the flow is thus encoded in a sequence $\Sigma=\{\sigma_n\}$ of symbols defined according to

$$
\sigma_n = \begin{cases} 
2 & \text{if } \theta_n \in \left[ -\frac{\pi}{4}; \frac{3\pi}{4} \right] \\
1 & \text{if } \theta_n \in \left[ -\frac{\pi}{2}; -\frac{\pi}{4} \cup \frac{3\pi}{4}; \frac{3\pi}{2} \right]
\end{cases}
$$

(1)

Each intersection of the trajectory with the Poincaré section is thus converted in a symbol, 2 or 1 according to the selected angle. With $U_r=2.09$ m s$^{-1}$, this partition gives no preferred symbol: both are equally selected. Since the two main peaks in the Fourier spectrum do not have the same size, this reminds us that a symbol does not exactly correspond to one mode. Indeed, the dynamics is locked on a given mode only when subsequences $\ldots 2222\ldots$ or $\ldots 1111\ldots$ are encountered. However when subsequences such as $\ldots 2121122\ldots$, the symbolic dynamics just evidences transitions between the two modes and it becomes rather difficult to associate 2 or 1 with a specific mode. The mean length $\bar{l}$ for subsequences $2^i$ ($1^i$) is 3.85 (3.55). This means that, in average, symbols (2 or 1) are repeated most often three or four times. The probability of lengths $l$ during which a symbol is repeated corresponds to the probability with which a given symbol is repeated $i$ times. It clearly appears that the short lengths (between 1 and 3 oscillations) are more numerous for both symbols [Fig. 7(a)] but when the probability is computed versus the time duration, the long sequences of repeated symbols significantly contribute to the dynamics [Fig. 7(b)]. To sum up, the events corresponding to a symbol repeated during a long time duration are quite rare, but due to their length they contribute for a significative part of the time series.

Using $N=15\,000$ points in the Poincaré section, that is, 15,000 oscillations, the probabilities for various symbolic sequences can now be estimated. Many levels of investigation may be used according to the length $n$ of symbol sequences considered. At the first level, only the 1-symbol sequences
are considered and we thus have $q$ probabilities characterizing the dynamics, namely, $P_0, P_1, \ldots, P_{q-1}$ where $P_1$ is the probability of observing the symbol $i$. When the 2-symbol sequences are considered, $q^2$ probabilities describe the dynamics. For instance, $P_{12}$ is the probability of observing the sequence “12,” and so on. At the $n$th level, $q^n$ probabilities are used. The higher the level, the better the description of the dynamics. But to ensure that the statistics remain sufficiently well defined, the choice of $n$ is limited by the length $N$ of the set of intersections $\{P_i\}$. We limit the choice of $n$ as $q^n/N = 1\%$.4 In the present case the investigation was performed with $n=8$.

A statistical analysis is only possible through investigating the different probabilities for sequences of a given length $n$ occurring. The analysis may then provide a histogram giving such probabilities. In order to obtain the histograms, the different symbolic sequences are ordered as follows. The $q^n$ possible sequences composed of $n$ symbols chosen among $q$ different symbols, are indexed according to the natural order of the integer expressed in the $q$-basis but shifted by 1. For instance, in the case where binary symbolic dynamics is considered, sequence 222 122 is converted into 000 100 and is associated with the index $i=4+1$, 4 being the integer corresponding to the binary number 000100.

The histogram of the symbolic sequences realized by the dynamics is not flat (Fig. 8). This means that the underlying dynamics does not correspond to a white noise neither a complete symbolic dynamics.4 Obviously two main sequences are observed: $\Sigma_1=2222 2222$ and $\Sigma_{256}=1111 1111$. Since repeated symbols correspond to sustained modes, there is a preponderance for sustaining mode $f_2$ or mode $f_1$, alternatively. The sequence $\Sigma_{256}$ is slightly more often realized in agreement with the time durations during which symbol 2 (48%) and symbol 1 (52%) are observed. Then isolated from the “background” (probability greater than 0.017) the most probable sequences are

$$
\Sigma_{128} = 2111 1111 \quad \Sigma_{129} = 1222 2222,
$$

$$
\Sigma_{193} = 1211 1111 \quad \Sigma_{64} = 2122 2222,
$$

where $P_i$ is the probability with which each sequence $\Sigma_i$ is realized. Thus, it is found that the sequence $\Sigma_i=\Sigma_i$ is realized with a probability $P_i$ nearly equal to $P_i$. There is therefore a symmetry between the two symbols.

One of the main characteristic of the dynamics underlying our experiments is its ability to produce a competition between two modes that we converted into two symbols. In order to evidence that the dynamics privileges repetitions of symbols, a “transitional” symbolic dynamics is introduced as follows. What is encoded is now repetitions and transitions between two modes that we converted into two symbols. In order to evidence that the dynamics privileges repetitions of symbols, a “transitional” symbolic dynamics is introduced as follows. What is encoded is now repetitions and transitions from one symbol to the other which are encoded. The previous symbolic sequences are thus converted into new symbolic sequences according to

$$
\xi_i = R, \quad \text{if } \sigma_0 \sigma_{i+1} = 22 \text{ or } \sigma_0 \sigma_{i+1} = 11,
$$

$$
\xi_i = T, \quad \text{if } \sigma_0 \sigma_{i+1} = 12 \text{ or } \sigma_0 \sigma_{i+1} = 21.
$$

A probability density function is then computed for this new encoding (Fig. 9). The main peak is observed for $\Xi_1 = RRRR RRRR$, that is, for repetitions from one mode to the other. The tendency to observe a repeated symbol—sustained mode—is thus a dominant characteristic of this dynamics. Among the most often realized sequences, there are

$$
\Xi_{129} = TRRR RRRR,
$$

$$
\Xi_{65} = RTRR RRRR,
$$

$$
\Xi_{33} = RRTR RRRR,
$$

These most often realized sequences can be paired as follows. Let $\bar{2}=1$ and $\bar{1}=2$, that is, the complementary function $\bar{\Sigma}_i$ maps each of its symbols to the other ($2 \rightarrow 1$ and $1 \rightarrow 2$), then the complementary sequence $\bar{\Sigma}$ corresponds to the sequence of complementary symbols $\{\bar{\sigma}_i\}$. We have thus, $\Sigma_{128}=\Sigma_{129}$, $\Sigma_{193}=\Sigma_{64}$, $\Sigma_{253}=\Sigma_{4}$, and $\Sigma_{255}=\Sigma_{2}$. From the histogram, it is observed that

$$
P_{129} = 0.022 \quad P_{129} = 0.023,
$$

$$
P_{193} = 0.018 \quad P_{64} = 0.019,
$$

$$
P_{253} = 0.018 \quad P_{4} = 0.017,
$$

$$
P_{255} = 0.022 \quad P_{2} = 0.023,
$$

FIG. 8. Probability density functions of the symbolic sequences. Sequences corresponding to repeated symbols ($i=1$ and $i=256$) are obviously the most probable.

FIG. 9. Probability density function of the different symbolic sequences built on the transitional symbolic dynamics.

$$
\Sigma_{253} = 1111 1122 \quad \Sigma_{4} = 2222 2211,
$$

$$
\Sigma_{255} = 1111 1112 \quad \Sigma_{2} = 2222 2221.
$$
\( \Xi_{17} = \text{RRRT RRRR} \),
\( \Xi_{9} = \text{RRRR TRRR} \),
\( \Xi_{5} = \text{RRRR RTRR} \),
\( \Xi_{3} = \text{RRRR RRTR} \),
\( \Xi_{2} = \text{RRRR RRRT} \),
\( \vdots \)
which are the cyclic permutation of an isolated transition. This is a signature of sequences corresponding to more than eight repeated symbols (eight oscillations). There are also
\( \Xi_{193} = \text{TTRR RRRR} \),
\( \Xi_{161} = \text{TRTR RRRR} \),
\( \Xi_{97} = \text{RTTR RRRR} \),
\( \Xi_{81} = \text{RTRT RRRR} \),
\( \Xi_{49} = \text{RRTT RRRR} \).

The relative preponderance of these sequences reveals that once the dynamics realizes a symbol—visits a mode—the destabilization of that mode is not necessarily to switch to the other mode for a long period, but to quickly return to the same mode. Subsequence “TT” means two successive transitions, that is, 121 or 212. Subsequence “TRT” means 1221 or 2112, that is, a symbol repeated twice. Thus, the dynamics is mainly governed by the competition between two modes, which tend to be sustained. In other words, once a mode is locked, it tends to exclude the other. Such a feature explains the length of the bursts observed in the spectrograms [Fig. 3(b)] when two successive transitions are almost not seen in a spectrogram since over a time duration less than the time window over which the frequency is estimated. Sequences \( \Xi_{193}, \Xi_{97}, \) and \( \Xi_{49} \) explain why one might have the impression that a mode can be sustained over a long time duration. In fact, two successive transitions often interrupt repetition of a symbol but they are hardly seen, particularly with a frequency analysis, since very short.

A plane projection of typical trajectories associated with few realizations of each symbols (Fig. 10) show that mode 1 (\( f_{1} = 23.2 \) Hz) has an amplitude larger than the amplitude of mode 2 (\( f_{2} = 31.1 \) Hz). This is in agreement with the fact that the dynamics could be structured around a fixed point of the focus type. In such a case, the oscillations have a time period which increases as a monotonic function of the distance to the fixed point (amplitude). The two types of transitions [Fig. 11] confirm the fact that typically this switching process from one mode to the other is performed in a single oscillation (between two successive intersections with the Poincaré section).

Such an analysis can also help us to investigate how the distribution between symbols 1 and 2 evolves versus a parameter like the external velocity \( U_{e} \). For instance, with

**FIG. 10.** (Color online) Plane projections of typical trajectories associated with the two different symbols: mode 1 (a), mode 2 (b).

**FIG. 11.** (Color online) Plane projection of few typical trajectories associated with a transition from mode \( f_{2} \) to mode \( f_{1} \) (a) and from mode \( f_{1} \) to mode \( f_{2} \).
$U_c = 1.27 \text{ m s}^{-1}$, it is found that the angular interval $\theta_n \in [-\pi/2; -\pi/4] \cup [3\pi/4; 3\pi/2]$ is mostly visited. Most oscillations are therefore encoded by 1, which means, that it is easy to obtain very long sequences of 1s. This confirms the fact that the spectral analysis reveals a single main peak (Fig. 2).

V. CONCLUSION

The dynamics underlying an open flow over a cavity was investigated from the measurement of one component of the velocity using a LDV technique. As observed in other similar configurations reported in the literature, a nonlinear competition between two modes is investigated using tools borrowed to the nonlinear dynamical systems theory. From a plane projection of the phase space—reconstructed with the principal components computed from the component of the velocity—an angular first-return map allows to define a symbolic dynamics. Two symbols are used to distinguish the two modes in competition. From this map the two modes are found to be mainly exclusive, that is, a single mode drives the dynamics at a given time. A statistics on the symbolic sequences exhibits that the underlying dynamics is not a white noise (otherwise, a flat histogram in the symbolic sequences would have been obtained), neither a pure deterministic system characterized by a complete symbolic dynamics. It is rather a dynamics resulting from the superposition of a deterministic component (it exists toroidal structure in the phase space) and a stochastic process (as exhibited by the cloud of points observed in the Poincaré section). With the help of the symbolic dynamics the switching process between the two modes was found to occur very often. The time scale of these switches is so small that it cannot always be detected by spectral analysis.