Abstract

We study the behavior of sources and sinks in an experiment where left and right traveling waves are excited on the free surface of a fluid that is heated by a wire stretched under the surface. To study phenomena very close to threshold the experiment is made very long (2 m), while a precise linear map of the free surface is provided by a scanning device. The observed dynamical behavior of the sources is in agreement with a recent prediction based on the coupled complex Ginzburg–Landau equation [Physica D 134 (1999) 1]. However, the behavior of the sinks disagrees with it. When sources turn unstable they eject holes which are believed to be of homoclinic type.

1. Introduction

Weakly nonlinear waves in one dimension which spring from a supercritical bifurcation are generically described by the complex Ginzburg–Landau equation [1]:

\[ \tau_0 (\partial_t A + s_0 \partial_x A) = \varepsilon A + \xi_0^2 (1 + i c_1) \partial_x^2 A - g_0 (1 - i c_3) |A|^2 A, \]

where the linear growth factor \( \varepsilon \) is the reduced excitation strength. At onset \( \varepsilon = 0 \) no waves exist and for \( \varepsilon > 0 \) a bifurcation has occurred to traveling waves \( A(x,t) = e^{i(q_0 x - \omega_0 t)} \). Eq. (1) then describes the dynamics of the complex amplitude \( A(x,t) \) on slow time \( (\varepsilon^{-1} t) \) and large length \( (\varepsilon^{-1/2} x) \) scales. The wavenumber \( q_0 \) and the frequency \( \omega_0 \) of the fast wave motion are determined by the details of the actual system whose physical space and time scales are set by \( \xi_0 \) and \( \tau_0 \), respectively. Eq. (1) describes a wide range of nonlinear wave phenomena for which predictions can be reached without exact knowledge of the parameters. Moreover, it is often possible to learn about the coefficients directly from experimental observations. In rare cases, the coefficients can be derived theoretically from first principles. Our interest in Eq. (1) stems from predictions that can be made for wave modulations in the form of coherent structures.
emitted from it, and a sink if it absorbs waves. The dynamical behavior of sources and sinks may be described by a pair of coupled complex Ginzburg-Landau equations, one for the amplitude \( A_L(x,t) \) of the left traveling waves, and one for the right going \( A_R(x,t) \):

\[
\begin{align*}
t_0 \left( \partial_t A_R + s_0 \partial_x A_R \right) &= \epsilon A_R + \frac{\epsilon_0^2}{i} (1 + ic) A_R^3 - g_0 (1 - ic) A_R^3 - g_2 (1 - ic) |A_R|^2 A_R, \\
t_0 \left( \partial_t A_L - s_0 \partial_x A_L \right) &= \epsilon A_L + \frac{\epsilon_0^2}{i} (1 + ic) A_L^3 - g_0 (1 - ic) A_L^3 - g_2 (1 - ic) |A_L|^2 A_L,
\end{align*}
\tag{2}
\]

where \( g_2 \) reflects the coupling between the left and right going amplitudes. Of course, as the envelope equation (2) is ignorant of \( g_0 \) and \( s_0 \) of the waves, the definition of sources and sinks which are solutions of Eq. (2) involves the group velocity of the waves, rather than their phase velocity. Whereas the group velocity \( \omega_g \) can be removed from Eq. (1) in the reference frame moving with \( \omega_g \), this is no longer possible for Eq. (2).

The unavoidable emergence of the group velocity signifies the non-uniform validity of the coupled amplitude equation (2) in \( s \). In other words, the variation of \( \omega_g \) may no longer be slow in the rest frame moving with \( \omega_g \), it is no longer possible for Eq. (2).

In numerical simulations these unstable sources were indeed observed [9]. It was also found that the width of sources scales differently with the driving strength \( \epsilon \) above and below \( \epsilon_{c0}^L \). For \( \epsilon > \epsilon_{c0}^L \), the source width \( w \) seems to diverge as \( w \sim (\epsilon - \epsilon_{c0}^L)^{-1} \), with \( \epsilon_{c0}^L > 0 \), but for \( \epsilon < \epsilon_{c0}^L \), the average width of the unstable sources diverges as \( w \sim \epsilon^{-1} \). The width of the sinks, on the other hand, was always found to grow as \( \epsilon^{-1} \). The verification of these remarkable predictions is the principal goal of this paper.

Another key result of [9] is concerned with the uniqueness of sources and sinks. It was predicted that sources come in a discrete set, i.e. no two sources can coexist with only slightly differing properties. Specifically, there is a unique source with zero velocity. Sinks, on the other hand, would come in a two-parameter family. The power of these predictions is that they were reached using counting arguments that are insensitive to the value of the parameters in Eq. (2). By doing many experiments and by studying the emerging constellation of sources and sinks, we provide an anecdotal test of this prediction.

Counterpropagating traveling waves offer an appealing test of the amplitude description as sources and sinks are an intrinsic property that does not depend on
be insensitive to the details of the experiment. The experiment consists of a long heated wire that is stretched under the free surface of a fluid. The control parameter is the heat $Q$ dissipated in the wire. Through a combination of gravity- and surface tension-induced convection, traveling waves emerge on the free surface at $Q = Q_c$. The interest in this nonlinear system grew from experiments in which a laser beam that is reflected off the fluid surface onto a position sensitive device. Both laser and position detector were mounted on a computer-controlled cart which traveled (velocity $0.3 \text{ m s}^{-1}$) on precision machine polished stainless steel rods. The smallest surface wave amplitude that could be measured this way is a mere $0.5 \mu \text{m}$. The precision and linearity of this detection technique allowed us to move very close to threshold.

The signals of the reciprocating scanning device were assembled in scan lines $s(x,t)$ which were band-filtered around the apparent wave frequency and Hilbert transformed to yield local modulus $|A_m(x,t)|$ and phase $\phi(x,t)$ information, $s(x,t) = |A_m(x,t)| \text{e}^{i \phi(x,t)}$. The local wavenumber was computed from the slowly varying phase, $q(x,t) = \partial \phi(x,t)/\partial x$. To improve the signal to noise ratio, a running average over a time interval of $10 \text{ s}$ was performed. The finite scanning velocity necessitated a small correction for the Doppler effect. Another consequence of the finite scanning velocity is that information about the wave frequency is more difficult to come by, especially when large $x$-intervals are scanned. Both the control of the power $Q$ and of the scanning system are automated, which allowed unattended experiments that lasted several days.

Several sources and sinks can be recognized in Fig. 2 where we have plotted an $x$-$t$ diagram of the local wavenumber. In this experiment the heating power was at $t = 0$ quenched from zero to $Q = Q_c(1 + \epsilon)$, with $\epsilon = 0.29$. After a short while, left and right traveling waves emerge which are separated by sources and sinks. Before this time the local wavenumber

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1 The brand name of the oil is Tegiloxan 3, produced by Goldschmidt AG (Essen, Germany). It has (at 21 °C) viscosity $\nu = 3.397 \times 10^{-3} \text{ m}^2 \text{s}^{-1}$, density $\rho = 892.4 \text{ kg m}^{-3}$, surface tension $\sigma = 38.3 \times 10^{-2} \text{ N m}^{-1}$, with temperature coefficient $\Delta \sigma/\Delta T = -0.7 \times 10^{-5} \text{ N m}^{-1} \text{°C}^{-1}$, and a refractive index $n = 1.395$. 2 The subscript ‘m’ distinguishes this amplitude $A$ from the solution of Eq. (2).
Fig. 1. (a) Schematic view of the heated wire experiment. A thin (0.2 mm diameter) wire (1) is stretched beneath the free surface of a fluid (depth $h = 2$ mm). When it is heated by sending an electrical current through it, surface waves are excited. The slope of the waves is measured by reflecting a laser beam off the surface onto a position sensitive detector (PSD) (3). The laser and the PSD are mounted on a cart (2) that rides on precision steel rods (4). (b) Cross section of the cell.

is undefined and determined by instrumental noise. The wavenumber is also undefined at the location of sources and sinks, which are zeros of the modulus $|A_m(x,t)|$.

3. Characteristic quantities

For a comparison with the theoretical predictions, it is necessary to determine for our experiment the characteristic time and length scales $\tau_0$ and $\xi_0$, the linear group velocity $s_0$ and the propagation speed $v_f$ of fronts with respect to the group velocity. First, however, we will discuss the expected range of validity of the amplitude description. From Eq. (1) it follows that near onset the modulus of the waves should depend on the reduced control parameter $\epsilon$ as $|A_m| \sim \epsilon^{1/2}$. Since our detection technique is extremely linear, the range in $\epsilon$ where this holds can be determined accurately. In Fig. 3 we show the measured $|A_m|^2$ as a function of $\epsilon$. The linear range extends to $\epsilon \approx 0.2$. The inset shows one period of the surface profile of a traveling wave. The profile is indistinguishable from a sine wave for $\epsilon = 0.12$, but develops higher harmonics for $\epsilon = 0.2$. We conclude that the amplitude description becomes questionable for $\epsilon \gtrsim 0.2$.

The characteristic time $\tau_0$ must be determined from the critical slowing down at $\epsilon = 0$. First, we will identify it with the time for the wave amplitude to saturate after the heating power is quenched from $Q = 0$ to $Q > Q_c$. In principle, by repeating this experiment for different $Q$, both $Q_c$ and $\tau_0$ can be determined. In Fig. 4 it is seen that this time $\tau_d$ scales very well with $\epsilon$ as $\tau = \tau_0/\epsilon$, with $\tau_0 = 120 \pm 5$ s. However, it is not the characteristic time scale of the system because the exponential rise of the amplitude to saturation occurs on a much shorter time scale. From Fig. 4 it follows that $\tau_d$ is determined by the delay before the amplitude starts to rise. We therefore take the exponential rise time $\tau = \tau_0/\epsilon$, with $\tau_0 \approx 18$ s as an estimate of the characteristic time. The emergence of the longer time scale $\tau_0$ in this experiment is puzzling. It is also remarkable that the linear behavior of $\tau_d^{-1}$ with $\epsilon$ extends to larger $\epsilon$ than the $\epsilon^{-1/2}$ behavior of the amplitude in Fig. 3.

The length scale $\xi_0$ was measured from spontaneously modulated waves which spawn from unstable sources at small $\epsilon$. At large values of the control parameter $\epsilon > 0.2$, the field is stable and the wavenumber variations that occur naturally are too small to be used for a measurement of $\xi_0$. The relation between amplitude and wavenumber needed for this technique [23] follows from substituting $A(x,t) = |A(q)| e^{i(qx - \omega t)}$ in Eq. (1), with the result

$$|A(q)|^2 \xi_0 = 1 - \frac{\epsilon^2}{2} (q - q_0)^2$$ or

$$|\tilde{A}(q)|^2 = 1 - \frac{\epsilon^2}{2} (q - q_0)^2. \quad (4)$$
Fig. 2. Space–time diagram of the wavenumber field $q(x,t)$ of traveling waves at $\epsilon = 0.20$. At $t = 0$, the power dissipated in the wire is quenched from $Q = 0$ to $Q = (1 + \epsilon)Q_c$. Before the emergence of waves, the wavenumber is undefined and determined by noise. The wavenumber is also undefined at the location of sources and sinks (which are indicated by So and Si, respectively). The extent of the $x$-axis is 0.597 m, the total time is 9828 s.

A typical modulated wavenumber field and the corresponding function $|\tilde{A}(q)|^2$ is shown in Fig. 5. From a measured surface slope field $s(x,t)$ we computed both the modulus $|A_m(x,t)|$ and wavenumber $q(x,t)$ field (such as shown in Fig. 5a). Holes in the amplitude field were avoided by only considering space–time regions where the amplitude was larger than a set threshold. Next a histogram of measured $q$ values was made. In each of its discrete bins, we computed the average over the measured $A_m$ values that were taken at the same $(x,t)$ location as the $q$’s belonging to that bin. In Fig. 5 we recognize the quadratic behavior $|\tilde{A}(q)|^2 = 1 - (q - q_0)^2$, with the correlation length $\xi_0$ determined by the curvature through $\xi_0 = \sqrt{\epsilon \alpha}$.

For several wave fields that were measured at different

Fig. 3. Testing the range of validity of the amplitude description. Dots: measured $|A|^2$ as a function of reduced power $\epsilon = Q/Q_c - 1$, with $Q_c = 2950$ mW; dashed line: linear fit. Inset, dashed line: surface profiles at $\epsilon = 0.12$, solid line: at $\epsilon = 0.20$. The amplitude of these profiles has been normalized.

Fig. 4. Time after which the waves reach full amplitude as a function of $\epsilon$. Inset: the amplitude of the waves at $\epsilon = 0.23$. We take the characteristic time $t_{0.5}$ as the rise time of the amplitude (the slope of the dashed line).
The correlation length $\xi_0$ was measured by constructing histograms such as shown in Fig. 5. The found values for $\xi_0$ fluctuated from run to run, but did not depend in a systematic way on $\epsilon$. From their mean value and variation we estimate $\xi_0 = (2.7 \pm 0.6) \times 10^{-3}$ m, with the $\xi_0$ in Fig. 5 on the high side. The small value of the correlation length implies that it is only very close to threshold that the length scales in our system will become larger than the wavelength.

The (nonlinear) group velocity was determined from the propagation of deliberate perturbations of the surface. We found that it has the same sign as the phase velocity, showing a weak $q$-dependence, $s(q) = 2.0 \times 10^{-4} + 5 \times 10^{-6}(q/2\pi - 53) \text{ m s}^{-1}$. Therefore, we assume $s_0 = 2.1 \times 10^{-4} \text{ m s}^{-1}$ for the linear group velocity.

The transition from convective to absolute instability at $\epsilon = \epsilon_{ca}$ is determined from the front speed $v_f$ relative to the group velocity $v_{gl} = \epsilon^{1/2} v_{gl0}$ as $\epsilon_{ca} = (s_0/v_{gl0})^2$. Therefore, a measurement of the front velocity provides an indirect estimate for $\epsilon_{ca}$. Fronts were made by quenching the heating power $Q$ to a finite value $Q = (1 + \epsilon)Q_c$ at $t = 0$. After a short while, waves invade the unstable surface in the form of fronts. The boundaries of these fronts travel with velocity $v_{gl} = s_0 \pm v_f$, respectively. Fig. 6 shows the evolution of the amplitude of the waves at $\epsilon = 0.051$. 

![Fig. 5](image1.png)  
*(a) Wavenumber field $q(x,t)$ at $\epsilon = 0.12$. The $x$-extent of the scan is 0.84 m, the total time is 9173 s. (b) Solid line: histogram of squared modulus $|A|^2$ vs. $q$ measured from the field shown in (a). Dashed line: fit of $|A|^2 = 1 - \xi_0^2(q - q_0)^2 / \epsilon$, with $\xi_0 = 3.0 \times 10^{-3} \text{ m}$. 

![Fig. 6](image2.png)  
*Fig. 6. Modulus $|A(x, t)|$ of a quenching experiment. The $x$-extent of the scan is 0.682 m, the total time is 5242 s. At $t = 0$ the power is quenched from $Q = 0$ to $Q = (1 + \epsilon)Q_c$, with $\epsilon = 0.051$ and several fronts evolve. The solid lines outline two fronts, the dashed lines indicate the median velocity between the fronts.*
This value appears to be below \( \epsilon_{\text{ca}} \) because the velocity of both fronts has the same sign as the group velocity. The result of several experiments, both at \( \epsilon < \epsilon_{\text{ca}} \) and at \( \epsilon > \epsilon_{\text{ca}} \) in \( \nu_{10} = 5.4 \pm 0.5 \times 10^{-4} \text{ m s}^{-1} \), from which \( \epsilon_{\text{ca}} \approx 0.15 \). By initiating fronts at different \( \epsilon \) and determining the reduced power where \( \epsilon = 0 \), \( \epsilon_{\text{ca}} \) could also be determined directly, \( \epsilon_{\text{ca}} = 0.1 \pm 0.02 \), which is smaller than the one found from \( \nu_{0} \) and \( \nu_{10} \).

We must realize, however, that the error in \( s \) which is smaller than the one found from \( \sigma \), is rather large and has a large effect on \( \epsilon_{\text{ca}} \). Improvements in the stability, control and automation of the experimental setup are underway to cure this problem. Finally, from the estimated values of \( \xi_{0} \) and \( \nu_{0} \) we can now reach an estimate for \((1 + c_{1}^{2})^{1/2} = 1.8\).

### 4. Sources

The width of a source and its position \( s_{0} \) were determined by fitting the measured modulus \( |A_{\text{m}}(x,t)| \) with the functional form

\[
S(x - s_{0}) = S_{1}(x - s_{0}) + S_{2}(x - s_{0}),
\]

\[
S_{2}(x) = A(1 + a \epsilon^{2 + c_{2}x})^{-1/2}
\]

are the solutions of the equation \( dS/dx = bS - \xi^{3} \).

The source width \( w \) is then defined as \( S_{2}(\pm w/2) = A/2 \). It turns out that the measured source profiles can be represented adequately by the function \( S(x) \), but we emphasize that our results do not depend on the precise functional form Eq. (5).

The dependence of the width \( w(t) \) of sources on the reduced control parameter \( \epsilon \) was measured in long experimental runs (each of them lasting several days) in which a source was located at large heating power \( \epsilon \approx 0.3 \), after which \( \epsilon \) was set at progressively smaller values until \( \epsilon = 0 \). At each set value of \( \epsilon \), the source was observed for several hours by scanning the fluid surface slope. A challenge of these experiments is to keep the relevant experimental circumstances constant. In many cases these experiments were frustrated by the disappearance of a slowly drifting source from the field of view.

From the width \( w(t) \) at discrete scan times \( t \) we computed the mean \( \langle w \rangle \) and the standard deviation \( \sigma = \langle (w - \langle w \rangle)^{2} \rangle^{1/2} \). The measured dependence of these two quantities on \( \epsilon \) is shown in Fig. 7, which is the result of several runs. At strong driving, the source is narrow and \( w \) approaches the wavelength \( \lambda = 2\pi/q \) of the traveling waves. For decreasing \( \epsilon \) it increases swiftly as \( \langle w \rangle \sim (\epsilon - \tilde{\epsilon})^{-1/2} \), with \( \tilde{\epsilon} \approx 0.14 \). At \( \epsilon \approx 0.15 \) a new scaling behavior starts with the source width diverging as \( \langle w \rangle \sim \epsilon^{-1} \).

With the emergence of a new scaling behavior at \( \epsilon \approx 0.15 \), Fig. 7 shows that the source width becomes erratic, with an rms variation \( \sigma \) that increases rapidly with decreasing \( \epsilon \). The nature of these fluctuations is evident from Fig. 8 that shows the \( x-t \) diagram of an isolated source at \( \epsilon = 0.11 \). Whereas sources for \( \epsilon > 0.15 \) are stationary, that of Fig. 8 is unstable. In a cyclic fashion the source grows wide, leaving an interval of near-zero wave amplitude behind in its core. When this unstable core becomes too wide, it...
Fig. 8. Space–time diagram of the wavenumber field $q(x,t)$ of an unstable source, $\epsilon_1 = 0.11$, the extent of the $x$-axis is 0.428 m, the total time is 10,485 s. The white box outlines a hole emitted by the source, whose profile is shown in Fig. 9.

spawns an outward spreading wave front. The phase of the waves in this front does not match with that of the plane waves outside the source and the resulting phase twist is carried away by a coherent structure which travels with a velocity which is comparable to the group velocity. We will discuss the nature of these structures in Section 5.

The position of the observed break in the scaling behavior of the source occurs at a reduced power $\epsilon$ which is slightly (but significantly) larger that $\epsilon_{ca}$ that was found directly from $x$–$t$ diagrams of fronts. Our observations strongly support the predictions of [9]. However, the dramatic nature of the source instability was not anticipated in [9]. In their numerical simulations the observed fluctuations of the source were small and disappeared altogether at asymptotically long times.

Although our measured source width seems to favor the inverse square root behavior over the dependence $\langle w \rangle \sim (\epsilon - \epsilon_{ca})^{-1}$, more results are needed to establish the scaling of the width in the absolutely unstable regime. However, we notice that the square-root behavior diverges at $\epsilon \approx \epsilon_{ca}$. This square root dependence is the normal way in which length scales of the modulations of nonlinear waves described by the complex Ginzburg–Landau equation diverge at onset. In this respect the $\epsilon^{-1}$ behavior of the source width in the convectively unstable regime is remarkable.

An $\epsilon^{-1}$ divergence of the source width was also found by Vince and Dubois [13], but the break in the scaling behavior at $\epsilon_{ca}$ and the transition to unstable sources was not observed. As the source can grow as wide as 0.3 m, and earlier experiments were only 0.6 m long, end effects cannot be excluded. Similar experiments on free surface convection, but now driven by a temperature difference between the sidewalls in a cell with smaller aspect ratio ($L/\xi_0 = 35$) have been reported by Garnier and Chiffaudel [22]. In these experiments the source and sink are pinned at each end of the cell. Near threshold, the width of the source was found to scale as $w \sim \ln (\epsilon - \epsilon_{ca})$. This particular scaling behavior was attributed to the convective/absolute transition at $\epsilon_{ca}$, which was anticipated but was not observed directly with no pattern being observed in the convective regime $0 < \epsilon < \epsilon_{ca}$. Presumably the length of the system was too small and end reflections were insufficient to sustain the growth of the convectively unstable mode.

5. Amplitude holes

A typical amplitude and wavenumber profile of a structure which is ejected by the unstable source of Fig. 8 is shown in Fig. 9. It propagates with a velocity $v_h \approx 1.9 \times 10^{-4}$ m s$^{-1}$ (which approximately equals $s_0$). A compression of the waves is accompanied by a dip of the local modulus of the wave field. Such
Fig. 8. Profiles of hole emitted from the unstable source of Fig. 8. Shown is the \((x,t)\) region in the box drawn in Fig. 8. (a) Wavenumber field \(q(x,t)\). (b) Modulus field \(-|A_m(x,t)|\) which is shown inverted for clarity. To help reading wavenumber and modulus differences off the vertical axis, the hidden line plot has been sectioned. Dashed line: typical wavenumber profile across the hole.

Other holes in Fig. 8 have, instead, a wavenumber dilation and travel with larger velocities \((v_h \approx 3.5 \times 10^{-4} \text{ms}^{-1})\). In Fig. 9 it can be observed that during the lifetime of the hole, the maximum of the local wavenumber is related to the minimum of the modulus.

In recent years, several types of coherent structures have been identified in the single complex Ginzburg-Landau equation \([2,4,24]\). Nozaki–Bekki holes \([2,4]\) are propagating solutions of Eq. (1) that connect two regions with different wavenumber. Their width diverges if these wavenumbers become equal. Clearly, for most of the structures observed in our experiments, the wavenumbers at both sides of the hole are equal. From the wavenumber profile in Fig. 9 and from the more qualitative gray-scale \(x-t\) diagram in Fig. 8 we conclude that the wavenumbers to the left and to the right of the hole are the same to within the experimental error. However, a hole that connects two regions with different wavenumber can be recognized in Fig. 2 where it originates from the annihilation of a source and a sink.

A family of holes that connect regions with equal wavenumber has recently been identified in \([24]\). Whilst Nozaki–Bekki holes are propagating solutions that exist in analytic form, these so-called “homo-clinic holes” need to be determined numerically. From extensive numerical simulations it was concluded that for \(c_{1,3} > 0\) homoclinic holes which come with a wave compression have a velocity smaller than the group velocity, whereas the dilation structures move faster than the group velocity \([25]\). Both compression and dilation holes can be recognized in Fig. 8 where the compression holes are white and the dilation holes are dark. Their velocity ranges from \(1.9 \times 10^{-4} \text{ms}^{-1}\) of a compression hole to \(3.5 \times 10^{-4} \text{ms}^{-1}\) of a dilation hole. The latter value is significantly larger than the measured group velocity. The trend of these velocities appears to be opposite to the one sketched for \(c_{1,3} > 0\).

According to \([25]\) a homoclinic hole is never stable: an initial phase twist is attracted to its one-dimensional unstable manifold, evolves along this manifold and eventually will perish either due to the creation of a defect, or will diffuse away. Both compressional and dilational homoclinic holes are a one-parameter family whose members can be accessed through the strength of the initial phase twist. For an unstable source these phase twists come with random sizes, resulting in many sorts of holes in Fig. 8. In order to prove that all holes in Fig. 8 belong to a one parameter family, we follow these holes from birth until they either die or disappear from the field of view, and plot in Fig. 10 the extreme \(q_{\text{max}}\) of the wavenumber \(q\) versus the minimum \(|A|_{\text{min}}\) of the modulus. The coordinates
Fig. 10. Scatter plot of the minimum of the modulus vs. the extreme (in \( x \)) of the wavenumber along each of the holes shown in Fig. 8. Both compression (large \( q \)) and dilation (small \( q \)) holes belong to a one-parameter family. The open circles refer to the hole in the box drawn in Fig. 8.

\((q_{\text{ext}}(t), |A|_{\text{min}}(t))\) of all holes are seen to trace out a curve that indicates a functional relation between \( q_{\text{ext}} \) and \( |A|_{\text{min}} \). The start of each curve is non-generic as it reflects the manner in which an initial condition is attracted to the unstable manifold of a hole. There is a striking resemblance between our Fig. 10 and Fig. 5b from [24].

The fate of wave compression holes at higher \( \epsilon \) is illustrated in the \( x-t \) diagram of Fig. 11 which was registered at \( \epsilon = 0.18 \), slightly above \( \epsilon_c \). In the typical lifecycle of such a structure it slows down, develops a zero of the modulus, suffers a phase slip and turns into a dilational structure which disperses subsequently. We believe that this dynamical behavior has not been observed earlier and it is interesting to learn if similar structures also exist in Eq. (1). In Fig. 11 several stationary sources can be recognized, which are all annihilated by sinks. The interaction with the holes causes a sink to wander erratically.

6. Sinks

Contrary to that of the sources, the width of sinks was predicted to diverge monotonically as \( \epsilon^{-1} \) [9]. However, the sinks found in our experiments are nar-
Fig. 12. Superposition of scans of the surface slope near a sink at $\epsilon = 0.15$. The position of the sink is indicated by the dashed line.

In our experiments, this implies that sinks always move towards regions with high wavenumber (low frequency). An inspection of Figs. 2 and 13 teaches that at large values of $\epsilon$ ($\epsilon \gtrsim 0.25$) this is not always the case and phase slips may occur across a moving sink. For smaller $\epsilon$, the sink motion always seems to be determined by phase matching. Examples are shown in Figs. 11 and 14, where the rightmost sink is seen to bend towards the (bright) high-$q$ region. Clearly, as was emphasized in [15], sinks whose dynamics is determined by phase matching are outside the scope of Eq. (2) because phase matching is an essential non-adiabatic effect that involves the fast time scales.

A detailed view of a sink’s phase matching is shown in Fig. 12 where we have superposed the surface slope near a sink at different times. From the almost perfect asymmetry of the surface slope at both sides of the sink it can be seen that the phase of the surface elevation is exactly opposite at both sides. Let us emphasize that this observation is made possible by the linearity of our measurement of the relevant order parameter.

7. Multiplicity

A salient prediction of [9] concerns the multiplicity of sources at $\epsilon > \epsilon_c^2$: if different sources emit different wavenumbers, they must come in a discrete set, characterized by their velocity. Moreover, a stationary source must be unique.

These predictions were confronted with our observations in many repeated runs, a few of which were already shown in Figs. 2 and 8 and a further illustration provided in Figs. 13 and 14 for $\epsilon = 0.25$ and 0.11, respectively. After quenching the power from zero to $Q = (1 + \epsilon)Q_c$, sources appear, which often emit waves with different wavenumber. In the case of Fig. 13 the wavenumber difference between dark and bright regions can grow as large as 5 m$^{-1}$.
A phase matching sink moves towards the neighbor-
ing source with the lowest frequency, which then leads
to annihilation of the source and sink. Asymptotically,
therefore, only states will survive where all sources
have the same wavenumber. However, this may not
be interpreted as a verification of the predicted source
uniqueness.

All our sources slowly drift, which indicates that
they are not pinned to non-uniformities in the experi-
ment. If sources would occur in a discrete set [9],
the drift velocities are expected to be discrete. We have not
been able to observe such discreteness. As the degree
of the discreteness is expected to depend on the pa-
rameters of the coupled Ginzburg-Landau equations
(2), which are largely unknown for our experiment, it
is not yet possible to either confirm or refute the pre-
dictions of [9].

8. Conclusion

We have reached favorable agreement between the
observed and predicted behavior of sources. Sources
are bound states of waves that are traveling outward
from it (according to their group velocity). The agree-
ment emphasizes the key role of the linear spreading
velocity in stabilizing sources. The observed instation-
arity of the sources is more dramatic than predicted in
[9], with the emergence of outward propagating holes.

The break in scaling behavior of the source width
and the onset of instationarity of the sources signi-
fies the transition from absolute to convective instabil-
ity at $\epsilon_c$. This transition was anticipated before [22],
but it is here demonstrated explicitly. We believe that
a sufficiently large size of the experiment is an im-
portant requirement to observe such a transition and
it would be interesting to follow the effect of an in-
creasing confinement.

The holes that were encountered in our experiments
are of a homoclinic type and most probably belong to
the one-parameter family of [24]. From their behavior
it is possible to infer the sign of $c_{1,3}$ [25]. All holes
found were narrow and non-adiabatic effects cannot
be ruled out. Further research on the nature and the
stability of these holes is needed.

Our results show that the coupled Ginzburg-Landau
equation (2) can be used to model experiments, despite
its non-uniform validity in $\epsilon$. In view of the successful
prediction for sources, the failure of the theory [9]
to predict the properties of sinks is remarkable. An
intriguing question is what causes this asymmetry.

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