Vertical convection regimes in a rectangular cavity: Prandtl and aspect ratio dependence

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Journées Convection Naturelle, Orsay, July 3, 2023
Introduction: 2D Differentially Heated Cavity (DHC)

History

- Ghelfgat (1999, 2017, 2020ab)
- Wang et al. (2021) [group of Lohse]
- ... etc.

Mainly Prandtl numbers 0.7 and 7
Introduction: 2D Differentially Heated Cavity (DHC)

Control parameters

Aspect ratio, $A = \frac{H}{W}$
Prandtl number, $Pr = \frac{\nu}{\alpha}$
Rayleigh number, $Ra = g \beta \Delta T H^3 / \nu \alpha$
Transient and steady state at small $Ra$

- Starting from rest with uniform temperature
- Switching on thermal forcing
- Steady state:
  - Detached top/bottom plumes
  - Stable stratification in the interior
Instability of steady state at \( Ra > Ra_c \)

\[ (x,z) = (0.1,0.1), A = 1.0, Pr = 0.71, Ra = 1.85 \times 10^8 \]

- Exponential growth, linear instability
- Non-linear saturation

**Our study**

- Linear stability of steady state for \( 0.5 \leq A \leq 2.0 \) and \( 0.1 \leq Pr \leq 4.0 \)
- \( Ra_c(Pr,A) \), shape of the leading mode
Numerical method

Nek5000

- Spectral Element code
- DNS/LES/RANS

Snek5000, Snek5000-cbox

- A Python package and a thin interface over Nek5000
- [https://github.com/snek5000/snek5000-cbox](https://github.com/snek5000/snek5000-cbox)
- Different kind of simulations
  - Non-linear
  - Linear
- Base state with Selective Frequency Damping (SFD)
Our procedure for each $A$ and $Pr$

1. Non-linear simulations
   Estimate $Ra_c$ 

2. Non-linear simulations with Selective Frequency Damping (SFD)
   3 base states at $3 \, Ra > Ra_c$ 

3. Linear simulations
   3 growth rates ($\sigma$) $\Rightarrow$ exact $Ra_c$ 

4. Decompose the leading linear mode (Hilbert transform)

\[
\theta'(x, z, t) = A_\theta(x, z) \cos(\omega t + \Phi_\theta(x, z)) e^{\sigma t}
\]
Base states from NL simulations (SFD): $A = 2.0$, effect of $Pr$

- Stable stratification $\Rightarrow$ Brunt-Väisälä frequency, $N_c = \sqrt{-\frac{g}{\rho_0} \frac{\partial \rho}{\partial z}}$
- Mean flow changes a lot with $Pr$:
  - Large scale circulation
  - Plume detachment
Base states from NL simulations (SFD): $Pr = 0.35$, effect of $A$

- Variation of \((\text{length of meandering}) / (\text{cavity width})\)
- $\uparrow A \equiv \downarrow Pr$. Increasing $A$ is equivalent to decreasing $Pr$. 
Diagnostics

Definitions

- $\omega$: frequency of perturbation
- Brunt-Väisälä frequency:
  \[ N_c = \sqrt{-\frac{g}{\rho_0} \frac{\partial \rho}{\partial z}} \]

Internal waves

- $\omega / N_c > 1 \Rightarrow$ no internal waves (fast)
- $\omega / N_c < 1 \Rightarrow$ internal waves (slow)
Oscillation frequency $\omega/N_c$ vs $Pr$

Internal waves
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- $\omega/N_c < 1 \Rightarrow$ internal waves (slow)

Strong variation with $Pr$ and $A$
- "Slow" and "fast" oscillations
- Sharp decrease for $Pr \simeq 0.65$
Oscillation frequency $\omega/N_c$ vs $Pr$ and $A$

$\Rightarrow$ different regimes in $(A, Pr)$ space

next slides: different results for $A = 1.0$
Regime FCC (Fast Cell Circulation), $A = 1.0$, $Pr = 0.1$, $\omega/N_c = 2.1$

- Fast ($\omega > N_c$)
- Large cell circulation
- Maximum where hot plume meets cold boundary layer
Regime SCC (Slow Circulation Cells), $A = 1.0, Pr = 0.35, \omega/N_c = 0.83$

- Slow ($\omega < N_c$)
- Maximum at detached plume
- Internal waves in the interior
- Global symmetric mode
Regime FACo (Fast Asymmetric Corner), $A = 1.0$, $Pr = 0.53$, $\omega/N_c = 3.4$

- Fast ($\omega > N_c$)
- Maximum at corner (boundary layer)
- Asymmetric
- No coupling between active regions
Regime SSCo (Slow Symmetric Corner), $A = 1.0$, $Pr = 0.71$, $\omega/N_c = 0.35$

- Slow ($\omega < N_c$)
- Maximum at detached plume
- Coupling through internal waves
- Symmetric global mode
Regime FSP (Fast Symmetric Plume), $A = 1.0$, $Pr = 2.8$, $\omega/N_c = 1.3$

- Fast ($\omega > N_c$)
- Maximum at detached plume
- Temperature gradients larger at top and bottom
- Internal waves at top and bottom
- Symmetric global mode
Regime FAP (Fast Asymmetric Plume), $A = 1.0$, $Pr = 4.0$, $\omega/N_c = 1.7$

- Fast ($\omega > N_c$) $\Rightarrow$ no internal waves
- No coupling between active regions
- Asymmetric mode
6 different regimes

- Different geometries of base state
- Fast or slow (with waves)
- Global (symmetric) or uncoupled local modes (asymmetric)
New instability conditions

\[ Pr \leq 1.0 \Rightarrow Re > Re_c \simeq 10^4 \]

\[ Pr > 1.0 \Rightarrow RePr^2 > \simeq 10^4 \]
Conclusions and perspectives

Conclusions

▶ Rich regime diagram in \((A, \ Pr)\) space
▶ Importance of internal waves for symmetry
▶ Global modes (symmetric) or local modes (asymmetric) in different regimes
▶ New criteria for unsteadiness

Perspectives

▶ Physical mechanism of individual regimes
▶ Shear or buoyancy?