CONVECTION THERMOELECTRIQUE
APPLICATIONS A LA MICROGRAVITE

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Journée thématique : Convection naturelle, aspects fondamentaux et applications
Introduction

• Heat transfer by natural convection in low gravity environment (G \(\sim 10^{-4}g, 10^{-2}g\))

• Vanishing Archimedean buoyancy to be replaced by another body force

• Candidate:
  • Dielectrophoretic force (Helmholtz-Kortweg force) in dielectric liquids: electric gravity

• Investigation of the simplest geometry: dielectric fluid bounded by two electrodes

• Linear stability analysis, DNS and experiments

• Interest for cooling microfluidic systems where strong electric fields can be achieved for low electric tension difference
DIELECTROPHORETIC FORCE

- A dielectric fluid of density $\rho$ and permittivity $\varepsilon$ in an electric field $\vec{E}$ is subject to a pondermotive force of density [Landau & Lifshitz : Electrodynamics of Continuous Media]:

$$\vec{f} = \rho_e \vec{E} - \frac{1}{2} \vec{E}^2 \vec{\nabla} \varepsilon + \vec{\nabla} \left[ \frac{1}{2} \vec{E}^2 \left( \frac{\partial \varepsilon}{\partial \rho} \right)_T \rho \right]$$

Coulomb Force

Dielectrophoretic Force

Electrostriction force

Charge injection, Electrophoresis, Electro-osmosis, Sedimentation, Screening

Dielectrophoresis : trapping, pumping
Thermoelectroconvection

Pressure modification of the fluid in strong fields
Important in two-phase fluids : cavitation, boiling

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Dielectrophoretic force and electric gravity

- The dielectrophoretic force can be split into a conservative force and buoyancy force:

\[
\vec{f}_{DEP} = -\frac{1}{2} \vec{E}^2 \nabla \varepsilon = \vec{\nabla} p_e + \delta \rho \vec{g}_e
\]

- Electric gravity:

\[
\vec{g}_e = -\nabla \Psi, \quad \Psi = -\frac{\varepsilon_1 e}{2 \rho_0 \alpha} \vec{E}^2
\]

- Magnitude of the electric gravity \( \sim V^2, d^3, \varepsilon_{ref}, 1/\rho_0, e/\alpha \)

- High frequency regime: \( f \gg \left( \frac{1}{\tau_v}, \frac{1}{\tau_{\kappa}}, \frac{1}{\tau_e}, \frac{1}{\tau_{\text{ion}}} \right) \) where \( \tau_{(v, \kappa)} = \frac{d^2}{(v, \kappa)}, \tau_e = \frac{\varepsilon}{\sigma E}, \tau_{\text{ion}} = \frac{d}{\mu E} \)
## Electric Gravity in Simple Geometries

<table>
<thead>
<tr>
<th>Electrode Type</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plane Electrode</td>
<td>$g_e \propto \frac{V^2}{d^3}$</td>
</tr>
<tr>
<td>Cylindrical Electrode</td>
<td>$g_e \propto \frac{V^2}{r^3}$</td>
</tr>
<tr>
<td>Spherical Electrode</td>
<td>$g_e \propto \frac{V^2}{r^5}$</td>
</tr>
</tbody>
</table>

The dielectrophoretic force in spherical geometry may model the mechanism of convection in planets [GEOFLOW, ISS, C. Egbers BTU, 2008-2016].

Thoroughly investigated by
Le Havre + Cottbus

Turnbull, Roberts, Stiles
ELECTRO-THERMOCONVECTION EQUATIONS

Electrohydrodynamic Boussinesq-Oberbeck approximation

\[ \nabla \cdot \vec{u} = 0 \]

\[ \frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla) \vec{u} = -\nabla \pi + \nu \Delta \vec{u} - \alpha \theta \vec{g}_e \quad \text{with} \quad \theta = T - T_{ref} \]

\[ \frac{\partial \theta}{\partial t} + (\vec{u} \cdot \nabla) \theta = \kappa \Delta \theta \quad \text{« Baroclinic » source of vorticity} \]

\[ \nabla \varepsilon(\theta) \vec{E} = 0 \]

\[ \nabla \times \vec{E} = 0 \quad \text{Thermoelectric coupling} \]

with the generalized reduced pressure

\[ \pi = \frac{P}{\rho} + \left[ \frac{\varepsilon_{ref} e \theta}{2 \rho} - \frac{1}{2} \left( \frac{\partial \varepsilon}{\partial \rho} \right) \right] \vec{E}^2 \]

\[ V(t) = V \sin(2\pi f t) \]

with \( f \gg (\tau_v^{-1}, \tau_k^{-1}, \tau_e^{-1}, \tau_{ion}^{-1}) \)

Electric field contribution to the pressure

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ELECTRO-THERMOCONVECTION EQUATIONS

CONTROL PARAMETERS

Prandtl number: \( Pr = \frac{\nu}{\kappa} \)

Electric Rayleigh-Roberts number: \( L = \frac{\alpha \Delta T g_e d^3}{\nu \kappa} \)

Thermoelectric coupling parameter: \( \gamma_e = e \Delta T \) where \( \varepsilon (\theta) = \varepsilon_{ref} [1 - e \theta] \)

METHOD OF INVESTIGATION

LSA: Linear Stability Analysis

DNS: Direct Numerical Simulations

Experiments on zero-g airbus (parabolic flight), Sounding Rocket
THERMOELECTRIC RAYLEIGH-BENARD CONVECTION (RBC)

Thermoelectric convection in a rectangular cavity

- Linear stability analysis to determine the threshold parameters
- Extensive DNS for quantitative characterization of higher modes states and heat transfer
- Experiments on parabolic flight campaigns

\[ d << (L_x, L_y) \]

\[ g_e = \frac{\varepsilon_1 e V_0^2}{\alpha \rho_0 d^3} \]

\[ \sim \sqrt{2V_0 \sin (2\pi f)} \]
In Microgravity (Ra=0), $L_c = 2128.7 > R_{ac} = 1708$

Critical states independent of Pr

Supercritical bifurcation: \[ \tau_0 \frac{\partial A}{\partial t} = \delta A + \xi_0^2 \frac{\partial^2 A}{\partial z^2} - |A|^2 A \]

with $\tau_0 = \frac{Pr + 0.5117}{19.65}$; $\xi_0 = 0.385$
THERMOELECTRIC RAYLEIGH-BENARD CONVECTION (RBC)

LSA : $L_c = 2128.7$ ; DNS : $L_c \approx 2131.9$

$L = 2200$  \hspace{1cm} $Q = 10$

$L = 4000$  \hspace{1cm} $Q = 2$

$L = 9000$  \hspace{1cm} $Q = 0.8$
Variation of the amplitude of modes: supercritical bifurcation

$$Nu \equiv \frac{hd}{k} = \frac{1}{L_x L_y} \int_{-L_y/2}^{L_y/2} \int_{-L_x/2}^{L_x/2} \left( Prw\theta - \frac{\partial \theta}{\partial z} \right) dx\,dy$$

convection

conduction
Experimental results in parabolic flight (zero-g phase) with BTU_Cottbus (Prof. C. EGBERS)

Horizontal Rectangular cavity

<table>
<thead>
<tr>
<th>Working fluid: Novec 7200 (20°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density ( \rho ) [kg.m(^{-3})]</td>
</tr>
<tr>
<td>Kinematic viscosity ( \nu ) [m(^2).s(^{-1})]</td>
</tr>
<tr>
<td>Thermal expansion coef ( \alpha ) [K(^{-1})]</td>
</tr>
<tr>
<td>Thermal diffusivity ( \kappa ) [m(^2).s(^{-1})]</td>
</tr>
<tr>
<td>Electric permittivity ( \varepsilon ) [F.m(^{-1})]</td>
</tr>
<tr>
<td>Coefficient. of thermal variation of permittivity ( e ) [K(^{-1})]</td>
</tr>
<tr>
<td>Prandtl number ( Pr ) [-]</td>
</tr>
</tbody>
</table>

\( V_{\text{peak}} = 1 \text{ kV}, L = 1000 \)  
\( V_{\text{peak}} = 2 \text{ kV}, L = 4000 \)  
\( V_{\text{peak}} = 3 \text{ kV}, L = 9000 \)  
\( V_{\text{peak}} = 5 \text{ kV}, L = 25 000 \)
• The DEP requires alternating tension with high frequency

![Diagram of Joule Heating in Thermoelectric RBC](image)

• The permittivity of the dielectric polar liquids is complex: \( \varepsilon = \varepsilon' - i\varepsilon'' \)

• The dielectrophoretic force averaged over a period is

\[
\begin{align*}
\langle f_{DEP} \rangle &= \frac{1}{4} \left\{ \varepsilon' \nabla \left( \hat{E} \cdot \hat{E}^* \right) - i\varepsilon'' \left[ \left( \nabla \hat{E} \right) \cdot \hat{E}^* - \left( \nabla \hat{E}^* \right) \cdot \hat{E} \right] \right\}; \hat{E} = -\left( \nabla \phi + i\nabla \psi \right)
\end{align*}
\]

• Joule dissipation in high Pr liquids:

\[
\langle P_D \rangle = \langle \vec{j}_D \cdot \vec{E} \rangle = \frac{1}{2} \omega \varepsilon'' \hat{E} \cdot \hat{E}^*
\]

with the displacement current \( \vec{j}_D = \frac{\partial \vec{D}}{\partial t}, \vec{D} = \varepsilon \vec{E} \)
The governing equations:
\[
\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla) \vec{u} = -\nabla \pi + \nu \nabla^2 \vec{u} + \frac{f_D}{\rho_0}
\]
\[
\frac{\partial \theta}{\partial t} + (\vec{u} \cdot \nabla) \theta = \kappa \nabla^2 \theta + \frac{P_D}{\rho_0 C}
\]

The increase of the thermocoupling parameter $\gamma_e$ above $10^{-2}$
- reduces the threshold of TEC induced by internal heating;
- increases the size of the convective cells induced by internal heating.
JOULE HEATING IN THERMOELECTRIC RBC

Perturbation velocity & temperature fields

\[
Pr = 181, \tan \delta = 0.00612, \Omega = 3.79 \times 10^{-6}
\]

\[V_{Ec} = 1262 \& k_{x,c} = 4.1 \text{ for } \gamma_c = 0.369\]

\[T_1 = T_2 : \text{Critical mode}\]

Four cells per wavelength!
THERMOELECTRIC CONVECTION IN A CYLINDRICAL CAPACITOR

Base state: conduction regime

\[ V_E = \frac{V_0}{V_i} = V_0 \left( \frac{\varepsilon_{\text{ref}}}{\rho \nu \kappa} \right)^{1/2} \]

\[ L = f(\eta) \ V_E^2 \ ; \ f(\eta = 0.5) = 0.0067 \]

Conductive state

The conductive state is invariant with respect to rotation and to translation along the axis away from the ends.
Linear stability: state diagram: \((\eta = r_1/r_2, L_c)\)

- \(L_c\) is independent on \(\text{Pr}\);
- \(L_c\) depends sensitively on \(\eta\) and on \(\gamma_e\)


DEP convection in Plane Geometry

Critical modes: Helical stationary vortices
Stationary Helicoidal vortices as predicted by the LSA

$V_E = 500$

The instability breaks the translation invariance along the axis and the rotation invariance.

At $z = 10$

Stationary defect due to the symmetry breaking

<table>
<thead>
<tr>
<th>$V_{E_c}$</th>
<th>Present (DNS)</th>
<th>Experiment (Meyer et al. 2017)</th>
<th>LSA (Meyer et al. 2017)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$450 &lt; V_{E_c} &lt; 500$</td>
<td>$732 &lt; V_{E_c} &lt; 916$</td>
<td>473</td>
<td></td>
</tr>
</tbody>
</table>
Weakly oscillating helical vortices

\[
V_E = 900
\]
WEAKLY OSCILLATING HELICAL VORTICES

$V_E = 1000$

$Q = 0.6$

Movie!
WAVY HELICOIDAL VORTICES

\[ V_E = 5000 \]

\[ Q = 30 \]

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Movie
THERMOELECTRIC CONVECTION IN A CYLINDRICAL CAPACITOR

$V_E = 500$  $V_E = 900$  $V_E = 1500$  $V_E = 4000$  $V_E = 5000$
The cylindrical annulus allows to reach much higher values of $Nu$ than the rectangular cavity.
PARABOLIC FLIGHT EXPERIMENTS

Experimental setup for cylindrical annulus

Geometry:
\[ R_1 = 5 \text{ mm} \; ; \]
\[ R_2 = 10 \text{ mm} \; ; \]
\[ l = 100 \text{ mm} \; ; \]
\[ \eta = 0.5 \; ; \Gamma = 20 \]

Parameter range: \[ 0 < V_p < 10kV \; \text{and} \; 0 < \Delta T < 20K \]

Sitat, 2000

Silicone oil AK5:
\[ \nu = 5.00e^{-6} \text{ m}^2/\text{s} \; ; \]
\[ \kappa = 7.74e^{-6} \text{ m}^2/\text{s} \; ; \]
\[ \alpha = 1.08e^{-3} \text{ K}^{-1} \; ; \]
\[ \epsilon = 1.07e^{-3} \text{ K}^{-1} \; ; \]
\[ \sigma = 2.7 \; ; \Pr = 6.4 \; . \]

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L = 17600
PARABOLIC FLIGHT EXPERIMENTS

Imposed voltage: $V_{\text{eff}} = 9 \text{ kV}$  

**Vertical columnal vortices**

**Inclined vortices**

**Shadowgraph images**

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Laser light sheet technique/ PIV

- Cross-correlation between experimental images in a time series
- Result is a 2D velocity field in the $r - z$ plane
- Calculated by Matlab with PIV-lab toolbox
- Tracer particles: Hollow Glass Spheres ($\rho = 1000 \text{kg} / \text{m}^3$, $\varepsilon = \ldots$)
- Particles may not interact with the electric field

M. Jongmanns et al., Micro Sci Technol. 2017

Meyer et al. 2016
Conclusion

• Numerical investigation of thermoelectric convection in planar and cylindrical capacitors

• Thermoelectric convection: stationary bifurcation

• Beyond the threshold: stationary defects, oscillations, chaotic patterns

• Computation of the heat transfer coefficient through the Nusselt number

• Experiments on zero-g airplane show thermoelectric convective structures well above the threshold

• More experiments with appropriate oil on parabolic flights and in laboratory to be close to the threshold and to measure heat transfer coefficient

• There are few assumptions that limit our approach: quasi-stationary approximation, Joule heating often neglected
THANK YOU FOR YOUR KIND ATTENTION