Resolving the stratification discrepancy of turbulent natural convection in differentially heated air-filled cavities – Part I: Reference solutions using Chebyshev spectral methods

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1. Introduction

We consider a Differentially Heated Cavity (DHC) for parameter values resulting in weakly turbulent flow. This configuration has been the subject of many studies in the past decades as it constitutes a canonical exercise for testing the performances of numerical codes and/or turbulence models. The early studies were performed in 2D but, with the continuously increasing capacities of computers, more and more are carried out in 3D (Dol and Hanjalić, 2001; Trič et al., 2000; Soria et al., 2004; Salat et al., 2004; Trias et al., 2007; Baragh and Davidson, 2007, for example). The laminar regime in a DHC was numerically explored in the late 1970s and during the 1980s, (Mallinson and de Vahl Davis, 1977; de Vahl Davis and Jones, 1983; Le Quéré, 1991; Fusegi et al., 1993) and these studies agree to a very large extent for values of the Rayleigh number up to 10^6, whatever the numerical method used.

The situation is not the same for transitional or weakly turbulent flows, that is for Rayleigh numbers of the order of 10^9–10^10. Most of the contributions used a RANS k–c approach with various turbulence models. Two 2D DNS (direct numerical simulation) solutions were provided, one by Paolucci (1990) for a Rayleigh number of 10^10 and one by Xin and Le Quéré (1995) for a Rayleigh number of 10^5. This comparison exercise pointed out that the RANS solutions displayed a large scattering. Later on, a “RANS reference solution” was defined as the average of the half contributions contained in the minimum interval. It is thus a compound solution of almost all the contributions as the contributions retained for one quantity (maximum velocity, for example) in the reference solution are not the same as those retained for another quantity (Nusselt number, for example). In particular, the “reference” dimensionless thermal stratification at the cavity center \( \frac{S}{\Delta T} = \frac{\Delta T \rho' c_p' u' T' z}{k} \) was set to \( S = 0.539 \) (Henkes and Hoogendoorn, 1995). It happens that this value was close to the...
thermal stratification equal to $(S) = 0.38$ predicted by Paolucci (1990), but it turned out that the integration time of Paolucci’s DNS was too short to reach the corresponding asymptotic flow regime. However several authors using 2D or 3D DNS or Large Eddy Simulations (LES) approaches (Xin and Le Quéré, 1995; Nobile, 2002; Sergent et al., 2003; Soria et al., 2004; Salat et al., 2004; Trias et al., 2010a) have reported dimensionless values of the stratification $(S)$ of about 1. In fact, $(S)$ taking a value close to 1 is in continuity with the value observed in the separated boundary layer regime for steady solutions with $Ra \sim 10^6$, as it is now well established (Janssen and Henkes, 1996; Le Quéré and Behnia, 1998).

More recently, new contributions using LES have provided numerical values of the thermal stratification which are in good agreement with experimental measurements of thermal stratification. For instance, Peng and Davidson (2001), dealing with the experimental cavity of Tian and Karayiannis (2000), have reported numerical results in good agreement with the experimental observations when performing a 3D LES using the measured temperature distributions on the top and bottom walls and either adiabatic or periodic Boundary Conditions (BCs) in the spanwise direction. This seems to indicate that combining 3D simulations with the measured temperature distributions on the top and bottom walls, is the key to the prediction of the thermal stratification with the measured temperature distributions on the top and bottom walls and either adiabatic or periodic BCs in the spanwise direction. This seems to indicate that combining 3D simulations with the measured temperature distributions on the top and bottom walls, is the key to the prediction of the thermal stratification in the cavity core. Dol and Hanjalić (2001) have also considered the measured temperature on the cavity walls as Dirichlet BCs when performing 2D and 3D RANS with either a second moment closure model or a modified $k$-$c$ model. In their study, the rear and front walls are either passive or thermally controlled but an unusual condition has been applied to the horizontal walls: an isothermal cold bottom wall and an isothermal hot top wall have been used. They have reported complete 2D and 3D comparisons with the experimental data available for the different configurations considered. However, their results have proved to be very dependent on the turbulence model and their comparisons with experimental data are not completely satisfactory. The persistence of the variety of predicted flow structures may lead to the occurrence of multiple solutions exhibiting different symmetries, as it has been shown in the Rayleigh number range $10^2-10^6$ in a cubic cavity (de Gassowski et al., 2006).

At the same time a joint research program has been set up in order to understand the unexplained discrepancy observed on thermal stratification in the cavity core. This joint team work is based on three different but complementary approaches: experimental measurements, 3D DNS and LES. A new experiment (Salat et al., 2004) has been built up to revisit the early experimental studies of Mergui and Penot (1997). Numerically, 3D DNS using spectral methods have been carried out along with 3D LES using finite volumes, for parameters corresponding to the experimental configuration.

This article, the first of three companion articles (Part I, Part II and Part III), is aimed at presenting a complete set of reference numerical results (DNS) in a DHC with idealized BCs (perfectly adiabatic BCs or measured temperature on the horizontal walls) for analyzing the influence of horizontal walls BCs on the flow structure. These two configurations combined with adiabatic BCs at the end walls could not have real physical meaning, especially in an air-filled cavity. However these results are meant to provide reliable data which will be the starting point of the end wall effects study in the Part II article. Moreover these two accurate solutions may also be used as an unphysical test case for numerical algorithms testing.

The Part II article (Sergent et al., 2013) defines several configurations with increasing levels of realism in the thermal BCs. The corresponding results of LES are compared. The increasing improvement in predicting the experimental thermal stratification in the cavity core is shown, which resolves the stratification paradox.

The Part III article (Xin et al., 2012) will show that the main physical phenomenon responsible for the temperature distributions on the cavity walls is the wall radiation.

The remainder of the present article is structured as follows: next section is devoted to the physical problem of interest, then the equations are given together with a brief description of the numerical methods. In Section 3, the results are presented laying the emphasis on the influence of the thermal distribution on the horizontal walls. Comparison is based on qualitative and quantitative results for time-averaged flow, turbulent transition, power spectra and turbulent statistics are given. The conclusions are given in Section 4.

2. Physical problem and mathematical formulations

2.1. Geometrical configuration and thermal characteristics

The geometrical configuration corresponds to an air-filled cavity of width $W$ in the $x$ direction, depth $D$, in the spanwise $y$ direction and height $H$ in the vertical $z$ direction as illustrated in Fig. 1. Its two opposite vertical walls in the $x$ direction are maintained at uniform but different temperatures $T_0$ at $x = 0$ and $T_f$ at $x = W$. Due to buoyancy, a fluid motion is induced in the cavity: it depends on the cavity geometry, the working fluid and the temperature difference, $\Delta T(T_f - T_0)$. The front and rear walls are considered to be thermally insulated. Depending on the studied configuration, the two other walls (top and bottom walls) are either thermally insulated or maintained at a constant temperature distribution (see Section 3.1).

In terms of dimensional analysis, the representative dimensionless parameters are the geometrical aspect ratios ($A_x = W/H = 1$, $A_y = D/H = 0.32$, $A_z = 1$), the Prandtl number, $Pr = \nu/\alpha$ ($Pr = 0.71$ for air) and the Rayleigh number, $Ra = g\beta\Delta T H^3/(\nu \alpha)$ ($Ra = 1.5 \times 10^8$), where $\beta$ is volumetric thermal expansion coefficient, $\nu$ gravity acceleration, $\nu$ kinematic viscosity and $z$ thermal diffusivity.

In the differentially heated cavity displayed in Fig. 1, the buoyancy force results in a clockwise circulation which consists, at large enough Rayleigh number, in a thin upward vertical boundary layer along the hot wall and a thin downward boundary layer along the cold wall. The vertical boundary layers are connected by horizontal flows which take place along the top and bottom walls. The cavity core is mostly at rest and experiences a thermal stratification with a quasi-linear temperature distribution in the vertical direction. This thermal stratification can thus be characterized by the vertical
temperature gradient at the cavity center: \( S = \frac{x}{\Delta T} (A_x/2, A_y/2, 0.5) \) where \( \theta = (T - T_0)/\Delta T \) is reduced temperature with \( T_0 = (T_h + T_c)/2 \). In turbulent regime, it is defined in time-averaged sense with \( \langle S \rangle = \frac{1}{T} \int_0^T \frac{x}{\Delta T} (A_x/2, A_y/2, 0.5) \) where the \( \langle \cdot \rangle \) denotes time averaging.

Time-averaged heat transfer at the cavity walls is defined by the following Nusselt numbers:

- **1D Nusselt numbers averaged along the vertical and horizontal lines at the cavity mid-depth (\( y = A_y/2 \)):**
  \[
  \langle Nu_{1D, \text{hot}} \rangle = \int_0^{A_x} - \frac{\partial (\theta)}{\partial x} (0, A_y/2, z) dz \quad \text{and} \quad \langle Nu_{1D, \text{cold}} \rangle = \int_0^{A_x} - \frac{\partial (\theta)}{\partial x} (A_x, A_y/2, z) dz
  \]
  \[
  \langle Nu_{1D, \text{bottom}} \rangle = \int_0^{A_x} - \frac{\partial (\theta)}{\partial x} (x, A_y/2, 0) dx \quad \text{and} \quad \langle Nu_{1D, \text{top}} \rangle = \int_0^{A_x} - \frac{\partial (\theta)}{\partial x} (x, A_y/2, 1) dx
  \]

- **2D Nusselt numbers averaged over the vertical and horizontal walls:**
  \[
  \langle Nu_{2D, \text{hot}} \rangle = \int_0^{A_y} \int_0^{A_x} - \frac{\partial (\theta)}{\partial x} (0, y, z) dy dz \quad \text{and} \quad \langle Nu_{2D, \text{cold}} \rangle = \int_0^{A_y} \int_0^{A_x} - \frac{\partial (\theta)}{\partial x} (1, y, z) dy dz
  \]
  \[
  \langle Nu_{2D, \text{bottom}} \rangle = \int_0^{A_y} \int_0^{A_x} - \frac{\partial (\theta)}{\partial z} (x, y, 0) dx dz \quad \text{and} \quad \langle Nu_{2D, \text{top}} \rangle = \int_0^{A_y} \int_0^{A_x} - \frac{\partial (\theta)}{\partial z} (x, y, 1) dy dx
  \]

Needless to note that adiabatic conditions along the top and bottom walls would imply zero Nusselt numbers there, although they are not equal to zero in the case of real experiments and in numerical simulations using measured temperature distribution on the top and bottom walls. Considering the symmetry properties of the investigated configurations (see Section 3.1), the time-averaged 2D Nusselt numbers of parallel walls will be equal, i.e. \( \langle Nu_{2D, \text{hot}} \rangle = \langle Nu_{2D, \text{cold}} \rangle \) and \( \langle Nu_{2D, \text{bottom}} \rangle = \langle Nu_{2D, \text{top}} \rangle \). Consequently, only one value for each pair is given in Table 2. The discrepancy between these two time-averaged 2D Nusselt numbers will be checked in Section 3.2 as a convergence criteria.

### 2.2. Governing equations

In the cavity is considered to be Newtonian and incompressible. Buoyancy-induced air flow is governed by the unsteady 3D Navier–Stokes under Boussinesq assumption in dimensionless form:

\[
\begin{align*}
\frac{\partial \psi}{\partial t} & = 0 \\
\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} & = -\frac{\partial p}{\partial x_i} + \frac{1}{Re Pr} \frac{\partial^2 u_i}{\partial x_j^2} + Pr \frac{\partial^2 u_i}{\partial z^2} \\
\frac{\partial T}{\partial t} & = 1 \frac{\partial^2 T}{\partial z^2}
\end{align*}
\]

Dimensionless Eq. (1) are obtained by using cavity height \( H \) as reference length and the convective velocity \( zRa^{1/2}/H \) as reference velocity. \( t \) denotes time, \( x_i \) denotes the coordinates \((x = (x, y, z))", u_i \) is the velocity component in the \( x_i \)-direction \((u_i = (u, v, w))", p \) is the pressure and \( \theta \), the reduced temperature, ranges from \(-0.5 \) on the cold wall to \( 0.5 \) on the hot wall.

In the present part of the three companions articles, the thermal BCs on the front/rear vertical walls \((y = 0, A_y)\) are set to be adiabatic \((\partial \theta/\partial y = 0)\). On the bottom/top horizontal walls \((z = 0, A_z)\), thermal BCs can be either adiabatic or of Dirichlet type (measured distributions). No-slip BCs are set for velocity field on all 6 walls.

### 2.3. Numerical methods

The DNS code is based on a time marching procedure and uses spectral collocation methods as spatial discretization. Time integration of the governing equations is performed through a second-order semi-implicit scheme. It combines a second-order backward Euler scheme with an implicit treatment for the diffusion terms and an explicit second-order Adams–Bashforth extrapolation for the nonlinear terms. This time scheme results mainly in Helmholtz equations which should be solved at each time step. Incompressibility is imposed by a projection method which retains second-order accuracy of the time integration (Guermond and Quartapelle, 1998; Achdou and Guermond, 2000).

Helmholtz equations for velocity components and temperature, and pseudo-Poisson equation for pressure correction are solved through total diagonalization of the discrete operators of second derivatives. Details of the method can be found in Xin and Le Quéré (1995) and Xin and Le Quéré (2002). The code has been validated previously for steady natural convection flow (de Gassowski et al., 2003) and 3D instability analysis (de Gassowski et al., 2006).

With Chebyshev collocation methods, grid points are defined in each direction \((x, y, z)\) by the Gauss–Lobatto distribution:

\[
\xi_i = \left(1 + \cos \left(\frac{i \pi}{N_c}\right) \right) \times \frac{A_c}{2}; \quad 0 \leq i \leq N_c
\]

where \( A_c \) is the cavity aspect ratio in direction \( \xi \).

In terms of wall units at \( Ra = 1.5 \times 10^9 \), the DNS grid results in \( \Delta x^+ \leq 0.2 \), \( \Delta y_{\text{max}}^+ \leq 0.2 \), \( \Delta z_{\text{max}}^+ \leq 0.4 \) in the PAC case (see Section 3.1 for definition).

### 3. Reference numerical results

#### 3.1. Investigated configurations

We are interested in two configurations, one is academic with perfectly adiabatic passive walls and another is more realistic with measured temperature distributions on the horizontal walls. They are defined as follows:

- The academic configuration is a **Perfectly Adiabatic Cavity (PAC)** with two opposite vertical active walls and four other (top, bottom, front and rear) adiabatic walls. This configuration should be the canonical problem for both experimental and numerical investigations. However, since adiabatic walls are almost impossible when working with air, it becomes an idealized configuration which is only suited for numerical approaches.

  - The more realistic configuration is named as **Intermediate Realistic Cavity (IRC)**. It is also defined with two opposite vertical active walls and adiabatic front and rear walls. On the top and bottom walls, temperature distributions are considered to be known and independent of \( y \) and \( t \). They correspond to the analytical fit of the measured temperature distributions along the centerline at \( y = A_y/2 \) (Salat, 2004):

\[
\begin{align*}
\theta_{\text{bottom}}(x, y) &= (0.5 - x) + 0.994 \left(\frac{x}{2}\right)^{1/4} \left(1 - 0.016 \left(\frac{x}{1 - 0.016}\right)^{0.8}\right) \\
\theta_{\text{top}}(x, y) &= -\theta_{\text{bottom}}(1 - x, y)
\end{align*}
\]
The IRC case relies on a close collaboration between experimental and numerical approaches and it is an important configuration toward resolving the stratification discrepancy of turbulent natural convection in differentially heated air-filled cavities.

Direct numerical simulations using spectral methods have been performed for the above configurations at $Ra = 1.5 \times 10^7$. They are aimed not only at understanding the corresponding flow regime but also at providing reference numerical solutions to both cases for the purpose of benchmark exercises. In a cubical PAC, Labrosse et al. (1997) and de Gassowski et al. (2003) have observed a first transition to unsteadiness at $Ra_{c1} = 3.2 \times 10^7$ and de Gassowski et al. (2003) have obtained several solution branches either steady or time-dependent up to $Ra = 10^8$. The Rayleigh number investigated is thus two decades higher than the first transition into unsteadiness in a cubic PAC. As can be seen below, flow is much more turbulent in the IRC than in the PAC, the DNS reference solutions of the IRC will be also used to demonstrate in the Part II article (Sergent et al., 2013) that the LES methodology is able to reproduce DNS results in a configuration which is more turbulent and closer to the experimental cell. The numerical parameters used in the simulations are reported in Table 1. As usual time integrations of the governing equations were started from a lower Rayleigh number and were carried out long enough for the flow regime and the turbulent statistics to become statistically established. In Table 1 the integration time is the time interval over which the governing equations have been integrated before starting the statistical sampling, expressed in units of dimensionless time. The averaging time is the period used for computing the turbulent statistics.

### Table 1

<table>
<thead>
<tr>
<th>Cavity type</th>
<th>Spatial resolution</th>
<th>Time step</th>
<th>Integ. time</th>
<th>Av. time</th>
</tr>
</thead>
<tbody>
<tr>
<td>DNSPAC</td>
<td>Perfectly adiabatic</td>
<td>$180 \times 100 \times 200$</td>
<td>0.005</td>
<td>655</td>
</tr>
<tr>
<td>DNSIRC</td>
<td>Intermediate realistic</td>
<td>$180 \times 100 \times 200$</td>
<td>0.002</td>
<td>612</td>
</tr>
</tbody>
</table>

3.2. Convergence tests

Before providing two benchmark solutions of low turbulent natural convection depending on the horizontal BC’s and demonstrating that academic configurations are not appropriate models for recovering experimental data in an air-filled cavity, several criteria of accuracy assessment are presented. Two causes of numerical errors may be put forward relating to the space and time convergence.

As said above, the present simulations of PAC and IRC have been performed with a Chebyshev spectral collocation approximation using 180 points or polynomials in the horizontal direction, 200 points in the vertical direction and 100 points in the spanwise direction. Past benchmarks (de Vahl Davis and Jones, 1983; Janssen and Henkes, 1996) have shown that for the types of flow under consideration, it requires at least five more points in each direction with a finite difference approximation to achieve the same accuracy. Achieving the accuracy of the present results with typical finite difference codes would thus have required on the order of 1000 mesh points per direction, which has not been done so far.

A further confirmation of the spatial accuracy of the present results can be obtained by looking at the decay of the spectral coefficients of the temperature or velocity fields. A close inspection of these coefficients shows that they decrease with increasing order, and the respective ratios of $10^5$ for IRC and $10^6$ for PAC are consistently observed. Past studies (Le Quéré and Alziary de Roquefort, 1985) have shown that such values guarantee spatial convergence of the Chebyshev expansion.

The second point relates to the integration time, and to the fact that we have integrated long enough in time to reach the true asymptotic turbulent regime. It is well known that the asymptotic regime is reached after the transient effects induced by the sudden step change in Rayleigh number have died away. To make sure this is indeed the case, the equations were integrated for around 600 time units before starting computing statistical quantities. As it takes approximately 40 time units for the fastest fluid parcels to perform one lap in the cavity, a time length of 600 thus corresponds to approximately 15 times this global circulation time. Statistics were performed over the next 200 time units which corresponds to approximately five times that global circulation time. To our knowledge computations thus long have not been carried out before, except in the work of Trias et al. (2007).

An indirect quantification of the time needed to obtain accurate statistics can be obtained from checking that the time averaged solution obeys to the well known symmetries, in particular the reflection symmetry about the mid-depth plane $y = A_i/2$, although the instantaneous solution does not and the global energy balance. The flow statistics reported hereunder (see Table 5) display this mid-depth symmetry to a very good approximation: the spatial deviation of the time-averaged transverse velocity field $\langle V \rangle$ to the theoretical value of zero has an averaged value on the mid-depth plane on the order of $10^{-6}$. Concerning the time-averaged global heat transfer $\langle N_{\text{tu}} \rangle$, the energy conservation between parallel walls is observed with a respective relative error of order of $10^{-5}$ and $10^{-4}$ for PAC and IRC. These two criteria confirm that the averaging time used was sufficient for computing accurate flow statistics.

### Table 2

<table>
<thead>
<tr>
<th>$\langle S \rangle$</th>
<th>$\langle N_{\text{tu}}(\text{hot-cold}) \rangle$</th>
<th>$\langle N_{\text{tu}}(\text{hot-\botop}) \rangle$</th>
<th>$\langle N_{\text{tu}}(\text{hot-cold}) \rangle$</th>
<th>$\langle N_{\text{tu}}(\text{hot-\botop}) \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DNSPAC</td>
<td>1</td>
<td>61.1</td>
<td>60.1</td>
<td>-</td>
</tr>
<tr>
<td>DNSIRC</td>
<td>1</td>
<td>59.3</td>
<td>57.9</td>
<td>12.1</td>
</tr>
</tbody>
</table>

**Fig. 2.** Vertical profile of time-averaged Nusselt number along the hot wall in the mid-depth plane ($y = A_i/2$). Comparison between PAC and IRC.
The combination of these two features, very large spatial resolutions and long time integrations, gives us confidence in the quality of the present results and allows us to claim their status of benchmark results.

3.3. Heat transfer

The PAC and IRC configurations differ only in the thermal BCs applied on the top and bottom walls. In IRC, these imposed temperature distributions result in a heat transfer between the fluid and the horizontal walls \( \frac{h_{\text{Nubottom}}}{h_{\text{top}}} \). This additional heat transfer does not modify significantly the global Nusselt number on the vertical walls \( \frac{h_{\text{Nu}}}{h_{\text{cold}}} \) nor the central stratification \( \langle S \rangle \) (see Table 2).

Despite the high value of the Rayleigh number, the overall Nusselt numbers along the vertical walls \( \frac{h_{\text{Nu}}}{h_{\text{top}}} \) still fit in both
cases the laminar $\overline{Nu} \sim \alpha_0 \cdot Ra^{1/4}$ relationship (Janssen and Henkes, 1996; Le Quéré and Behnia, 1998; Trias et al., 2007), observed for the $Ra$-range $10^6$–$10^9$.

This contradicts the study by Yu et al. (2007) which proposes that the Nusselt $1/3$ scaling behavior is valid for any configuration of laminar or turbulent confined natural convection (Rayleigh–Bénard convection as well as DHC). This statement is based on a scaling law establishment ($\overline{Nu} \sim \alpha_0 \cdot Ra^{0.31}$) for $Ra \leq 10^6$, which is obtained numerically for a PAC DHC with a 2D lattice Boltzmann approach.

It is noted that the centerline Nusselt numbers ($\overline{Nu}_{1/l}$) in both cases are slightly larger than the corresponding 2D Nusselt numbers. This is due to the presence of the front and rear walls as already noted by Tric et al. (2000) for steady flows in a cubic PAC. Although the global ($\overline{Nu}_{1(l)}$) is the same for both PAC and IRC cases, the mid-depth vertical profile of ($\overline{Nu}_{1(l)}$) is more uniform for IRC than PAC (Fig. 2). This comes from the important effect of the thermal BCs applied on the horizontal top and bottom walls. In fact, thermal boundary conditions modified the global flow structure and the turbulence level in the vertical boundary layers as can be seen in next section.

3.4. Time-averaged flow

The sensitivity of the flow structure to the thermal BCs applied on the bottom and top walls is shown in Fig. 3. Due to the local

![Fig. 5. Horizontal profiles of temperature ($\overline{\theta}$) and vertical velocity ($\overline{W}$) at various z positions and $y = A_l/2$. Comparison between PAC and IRC.](image-url)
heat transfer at the horizontal walls in the IRC, the horizontal flow along these walls is reinforced and experiences global reversal on the top and the bottom parts of the cavity. This reversal has also been reported by several numerical studies Dol and Hanjalić (2001), Peng and Davidson (2001), and Salat et al. (2004). They have shown that it was necessary to impose similar temperature distributions on the horizontal walls as that of experiments for reproducing the observed large flow reversal along the outer edge of the horizontal boundary layers (Tian and Karayiannis, 2000; Salat et al., 2004). This flow reversal does not exist in the PAC case, which displays the typical area of large eddies ejection instead (Xin and Le Quéré, 1995; Janssen and Henkes, 1996; Trias et al., 2007).

The IRC flow reversals create near the top and bottom walls two large areas which are more homogeneous in temperature than in the PAC. However the thermal stratification in the cavity core (approximately for $z$ between 0.35 and 0.65) is not changed by the modification of the thermal BCs on the top/bottom walls, as can be seen in Fig. 4.

The time-averaged characteristics of the velocity fields are gathered in Table 3. In both cases, the flow remains mainly two-dimensional in the mid-depth plane, where the spanwise velocity component $(V)$ values are weak. But for IRC, the magnitude of two horizontal velocity components $(U, V)$ is increased everywhere in the whole volume. This results from the aforementioned change of the flow structure, which affects not only the four boundary layers but also the cavity core.

Both cases differ substantially in the nearly motionless cavity core (Fig. 3). In PAC, the upward and downward boundary layers

![Fig. 6. Time-averaged wall shear stress on the hot vertical wall (left) and on the top horizontal wall (right) in the mid-depth plane (y = A_y/2). Comparison between PAC and IRC.](image)

![Fig. 7. Instantaneous fields of temperature $\theta$ (left) and horizontal velocity $u$ (right) in the vertical mid-depth plane (y = A_y/2). PAC (top) and IRC (bottom).](image)
entrain fluid through the whole cavity width respectively in the
bottom and top half parts of the cavity core. In IRC, the strong flow
reversals reduce by a factor two the height of the cavity core. The
upward vertical boundary layer receives fluid mainly from the fast
horizontal boundary layer up to the first quarter of the height
\((z \sim 0.25)\), then from the cavity core approximately up to \(z \sim 0.6\).

At mid-height of the cavity \((z = 0.5)\), the horizontal temperature
profile (Fig. 5) exhibits in both cases an undershoot temperature
region at the outer edge of the boundary layer, which is a typical
feature of thermal boundary layers in stratified medium (Yang
et al., 1972). This spatial oscillation is still present in the PAC case
at \(z = 0.7\), whereas it disappears at this height in the IRC case owing
to the flow reversal which interferes with the vertical boundary
layer.

In the PAC, the fluid carried by the vertical boundary layers im-
pinges the top or bottom wall and experiences several flow revers-
sals downwards and upwards before being dragged along towards
the opposite vertical wall. Fig. 3 displays the two separated (recir-
culating) horizontal boundary layers with successive detachments
and re-attachments in the top and bottom parts of the cavity. For
the sake of brevity, these horizontal fluid layers are called horizon-
tal separating fluid layers thereafter. The location of these

\[
\text{Fig. 8. Iso-surface of } \lambda_2, \text{ the second eigenvalues of } \Omega_{ik} \Omega_{kj} + S_{ik} S_{kj} \text{ (Jeong and Hussain, 1995), colored by the vorticity component } \omega_x. \text{ Left: PAC; right: IRC.}
\]

\[
\text{Fig. 9. Time series and normalized density power spectra of temperature at the cavity center (point A, see Table 4 for coordinates) in PAC (top) and IRC (bottom) cases.}
\]
separated regions can be evaluated from the horizontal distribution of the wall shear stress (Fig. 6) approximately at $x \sim 0.088-0.125$ for the first region and $x \sim 0.383-0.456$ for the second one. This figure also points out that the shear stress becomes slightly negative at $x \sim 0.21$, which is not visible in Fig. 3. As no separated region is present in the IRC case, a quasi-constant wall shear stress ($\tau_{wall}$) is generated in the center part of the horizontal walls ($0.3 \leq x \leq 0.8$).

The existence of the horizontal separating fluid layers in the PAC modifies the $U$-velocity vertical profile (Fig. 4) at the cavity mid-width ($x = Ax/2$) which is located downstream of the second re-attachment point. It also modifies the horizontal profiles at $z = 0.9$ of the temperature and the vertical velocity ($\langle W \rangle$) with the presence of the first separated region (Fig. 5).

### 3.5. Turbulent transition and instantaneous fields

Considering Fig. 5, we note for the IRC at $z = 0.7$ a thickening of the thermal and viscous boundary layers associated with a weaker counterflow by comparison with the PAC. Moreover the IRC $W$-velocity profile decreases nearly monotonically from a velocity maximum lower than the PAC peak. This indicates that the IRC cavity gives rise to an early turbulent transition of the vertical boundary layers. This is in good agreement with the vertical distribution of the mean wall shear stress (Fig. 6), which exhibits the classical laminar shape almost over the whole height of the PAC cavity, whereas the wall shear stress is reduced earlier in the downstream part of the IRC vertical boundary layer by the turbulent transition (Trias et al., 2010a).

The instantaneous fields of temperature and horizontal velocity component ($\langle U \rangle$) at the cavity mid-depth are displayed in Fig. 7. This figure confirms the very weak flow and the uniform thermal stratification of the PAC cavity core observed by Trias et al. (2007) in their three-dimensional simulations. Moreover the laminar–turbulent transition can be approximately located at a height $z \sim 0.8$, which also agrees well with their results. Unlike PAC, the IRC top wall is colder than the hot fluid coming from the upward vertical boundary layer and this results in a locally thermally unstable boundary layer, where small scale thermal plumes develop. These fluctuations are fed into the vertical downward boundary layer and trigger a much earlier transition to turbulence for IRC (with respect to PAC). It should furthermore be noted that the presence of vortices along the vertical walls indicates a transition location at around the mid-height of cavity ($z \sim 0.5$).

Fig. 8 depicts the coherent structures in the cavity, and especially those belonging to the boundary layers. It confirms the earlier turbulent transition occurring in IRC, as well as the fully turbulent regime of the downward part of the vertical boundary layers. It also shows that PAC exhibits a strong dependence on the spanwise direction, while still displaying the reflection symmetry. On the contrary, the IRC flow is much more homogeneous throughout the spanwise direction.

### 3.6. Power spectra

For both configurations, low frequency oscillations are observed in the cavity core as evidenced by the time evolution of temperature at the center and the corresponding power spectra shown in Fig. 9. The power spectrum is normalized such that the maximum of density is equal to one. The temperature fluctuations are weak and remarkably regular in PAC while of larger amplitude and more

<table>
<thead>
<tr>
<th>Table 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fundamental frequencies at the monitoring locations for PAC and IRC cases.</td>
</tr>
<tr>
<td>Point</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>Brunt-Väisälä frequency ($N = \sqrt{SPr/(2\pi)}$)</td>
</tr>
<tr>
<td>A</td>
</tr>
<tr>
<td>B</td>
</tr>
<tr>
<td>C</td>
</tr>
</tbody>
</table>

Fig. 10. Normalized density power spectra of temperature at two different locations (points B and C, see Table 4 for coordinates) in PAC (top) and IRC (bottom) cases. The Kolmogorov slope is reported for comparison (dashed line).
chaotic in IRC. However, a rather similar fundamental frequency is found in both cases (see Table 4) at around $f_A \sim 0.13$, which is in good agreement with the dimensionless Brunt–Vaïsälä frequency $N$ relating to the internal gravity waves.

The power spectra of the temperature are shown in Fig. 10 for two monitoring points. Point (B) is located in the horizontal separating fluid layer, whereas point (C) is located in the downstream part of the vertical boundary layer. For each cavity, the fundamental frequencies of both locations are of the same order. In the horizontal fluid layer (point B), the fundamental frequency is very weak (Table 4). It is representative of the large eddies ejected from the downstream part of the vertical boundary layers. As expected for the PAC (Xin and Le Quéré, 1995; Tian and Karayiannis, 2000), these large structures resulting from the separating fluid layer are characterized by a frequency smaller than $N$.

In the inner part of the boundary layer (point C), a high frequency peak is observed, corresponding to travelling waves. For the PAC, this peak at high frequency and its harmonics are visible in the power spectrum. This discontinuity in frequency of the spectrum is characteristic of a chaotic regime. Concerning the IRC, the power spectra are continuous for points (B) and (C) and exhibit a $-5/3$ slope for the highest frequencies, which indicates that the

Fig. 11. Isocontours of the turbulent kinetic energy $k = \langle u'^2 \rangle$ (top), the rms fluctuations of temperature $\theta_{\text{rms}}$ (middle) and the horizontal turbulent heat flux $\langle u'\theta' \rangle$ (bottom) in the vertical mid-depth plane $y = Ay/2$. PAC (left) and IRC (right).
Table 5
Maximum of \( \text{rms} \) fluctuations of temperature and velocity components for PAC and IRC cases.

<table>
<thead>
<tr>
<th>DNSPAC</th>
<th>U_{rms}</th>
<th>V_{rms}</th>
<th>W_{rms}</th>
<th>DNSIRC</th>
<th>U_{rms}</th>
<th>V_{rms}</th>
<th>W_{rms}</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_{\text{rms}} )</td>
<td>0.0226</td>
<td>0.0191</td>
<td>0.0257</td>
<td>0.0843</td>
<td>0.0306</td>
<td>0.0483</td>
<td>0.0675</td>
</tr>
<tr>
<td>( x = 0.0128 )</td>
<td>0.0218</td>
<td>0.0218</td>
<td>0.0049</td>
<td>0.0076</td>
<td>0.0272</td>
<td>0.0245</td>
<td>0.0128</td>
</tr>
<tr>
<td>( y = 0.1857 )</td>
<td>0.1650</td>
<td>0.1116</td>
<td>0.1650</td>
<td>0.1805</td>
<td>0.1546</td>
<td>0.2853</td>
<td>0.2305</td>
</tr>
<tr>
<td>( z = 0.9619 )</td>
<td>0.9700</td>
<td>0.9524</td>
<td>0.9263</td>
<td>0.6091</td>
<td>0.7129</td>
<td>0.9960</td>
<td>0.5392</td>
</tr>
</tbody>
</table>

Maximum of the \( \text{rms} \) fluctuations on the entire volume

<table>
<thead>
<tr>
<th>DNSPAC</th>
<th>U_{rms}</th>
<th>V_{rms}</th>
<th>W_{rms}</th>
<th>DNSIRC</th>
<th>U_{rms}</th>
<th>V_{rms}</th>
<th>W_{rms}</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_{\text{rms}} )</td>
<td>0.0375</td>
<td>0.0226</td>
<td>0.0191</td>
<td>0.0257</td>
<td>0.0843</td>
<td>0.0306</td>
<td>0.0483</td>
</tr>
<tr>
<td>( x = 0.0128 )</td>
<td>0.0218</td>
<td>0.0218</td>
<td>0.0049</td>
<td>0.0076</td>
<td>0.0272</td>
<td>0.0245</td>
<td>0.0128</td>
</tr>
<tr>
<td>( y = 0.1857 )</td>
<td>0.1650</td>
<td>0.1116</td>
<td>0.1650</td>
<td>0.1805</td>
<td>0.1546</td>
<td>0.2853</td>
<td>0.2305</td>
</tr>
<tr>
<td>( z = 0.9619 )</td>
<td>0.9700</td>
<td>0.9524</td>
<td>0.9263</td>
<td>0.6091</td>
<td>0.7129</td>
<td>0.9960</td>
<td>0.5392</td>
</tr>
</tbody>
</table>

Maximum of the \( \text{rms} \) fluctuations on the mid-depth plane (\( y = Ay/2 \))

<table>
<thead>
<tr>
<th>DNSPAC</th>
<th>U_{rms}</th>
<th>V_{rms}</th>
<th>W_{rms}</th>
<th>DNSIRC</th>
<th>U_{rms}</th>
<th>V_{rms}</th>
<th>W_{rms}</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_{\text{rms}} )</td>
<td>0.0374</td>
<td>0.0226</td>
<td>0.0075</td>
<td>0.0257</td>
<td>0.0789</td>
<td>0.0288</td>
<td>0.0422</td>
</tr>
<tr>
<td>( x = 0.0128 )</td>
<td>0.0218</td>
<td>0.2350</td>
<td>0.0049</td>
<td>0.0076</td>
<td>0.0245</td>
<td>0.0245</td>
<td>0.0128</td>
</tr>
<tr>
<td>( z = 0.9649 )</td>
<td>0.9704</td>
<td>0.9045</td>
<td>0.9263</td>
<td>0.6470</td>
<td>0.7545</td>
<td>0.9960</td>
<td>0.6319</td>
</tr>
</tbody>
</table>

Fig. 12. Horizontal profiles of the \( \text{rms} \) fluctuations of temperature (\( h_{\text{rms}} \)) and vertical velocity (\( W_{\text{rms}} \)) at various \( z \) positions and \( y = Ay/2 \). Comparison between PAC and IRC.
regime if fully turbulent in the vertical boundary layer. The order of magnitude of $f_c$ corresponds to the base frequency of the temperature fluctuations measured by Tian and Karayiannis (2000).

3.7. Second-order statistics

3.7.1. Qualitative analysis

It has been shown in Section 3.4 that in both configurations, the turbulence level is too low to impact the global Nusselt number ($N_{\text{Nu}_2}$): it keeps a scaling law which is characteristic of the laminar regime. In fact the major part of the cavity flow remains laminar mainly in the core. This can be observed in Fig. 11 displaying the spatial distributions of the turbulent kinetic energy ($k$), the $\text{rms}$ fluctuations of temperature ($\theta_{\text{rms}}$) and the horizontal turbulent heat flux ($u'\theta'$) in the mid-depth plane. In both cases a centro-symmetry with respect to the cavity center is exhibited. However the PAC spatial distribution of the fluctuations differs substantially from IRC. In PAC, the turbulent fluctuations are concentrated in the horizontal separating fluid layers. This does not agree with the recent results of Trias et al. (2010b), but it is explained by the fact that they considered a cavity of aspect ratio 4 which does not allow for the existence for a clearly defined detached region. The contour plot of ($u'\theta'$) shows the mixing effect of the successive detachments and re-attachments of the horizontal boundary layers, which make the top and bottom areas more uniform in temperature than the stratified core. In IRC the turbulent fluctuations are located along

Fig. 13. Horizontal profiles of the $\text{rms}$ fluctuations of the horizontal velocities $U_{\text{rms}}$ and $V_{\text{rms}}$ at various $z$ positions and $y = A_y/2$. Comparison between PAC and IRC.
the four walls and predominantly along the vertical isothermal walls. The comparison with the experimental data Tian and Karayiannis (2000) shows that the IRC configuration allows one to reproduce the spatial distributions of the turbulent intensities. The dissimilarity of the $k$ and $\theta_{rms}$ is indeed also observed experimentally as well as the abrupt growth located around the first quarter downstream of the vertical boundary layers. This height corresponds to the separating point between the horizontal flow reversals and the vertical boundary layers (Fig. 3).

3.7.2. Quantitative analysis

A summary of the maximum values of the Reynolds stresses is given in Table 5, which confirms the different turbulent levels induced by the BCs of PAC or IRC. Only $U_{rms}$ keeps a similar order of magnitude in both cavities. As PAC exhibits fluctuations related to the horizontal separating fluid layers, all the velocity $rms$ fluctuations maxima have the same order of magnitude. They are located close to the top-left or bottom-right corners, and in the mid-depth vertical plane except for $V_{rms}$, which is very weak in this plane. On the contrary, the IRC $rms$ fluctuations are characteristic of a wall turbulence, with a vertical component ($W_{rms}$) larger than the two other components, which have the same order of magnitude. The $W_{rms}$ and $\theta_{rms}$ maxima are located around the turbulent transition point of the vertical boundary layers.

3.7.3. Horizontal profiles in the hot boundary layer

In order to describe in details the PAC and IRC spectral solutions, the horizontal profiles of temperature and velocities $rms$ fluctuations at different heights are plotted in Figs. 12 and 13. There are no visible $rms$ fluctuations upstream from the PAC vertical boundary layers which remain laminar. But at $z = 0.9$, two peaks on the $rms$ profiles are observed: one in the downstream corner of the vertical boundary layer, and another resulting from the first recirculating region ($x > 0.125$). On the contrary, the fluctuations exist over the whole height of the IRC vertical boundary layers. Their peak intensities increase until the turbulent transition height ($z \leq 0.5$). At $z = 0.7$, the profiles do not decrease strictly monotonously: two successive inflection points are noticeable for the $\theta_{rms}$ or $W_{rms}$ profiles. This feature has been also observed experimentally by Tian and Karayiannis (2000). Fig. 13 shows that at this height the peak of $U_{rms}$ and $V_{rms}$ is smoother or even becomes a plateau.

4. Conclusion

Three-dimensional direct numerical simulations of a buoyancy-driven flow in a differentially heated air-filled ($Pr = 0.71$) cavity of aspect ratios ($A_x = 1, A_y = 0.32$) at a Rayleigh number equal to $1.5 \times 10^9$ have been presented. Two simulations have been performed using either adiabatic conditions (PAC) or experimentally measured temperature distribution (Salat, 2004) on the top and bottom walls (IRC), while the front and rear walls were assumed to be adiabatic. The present simulations of PAC and IRC have been performed with a Chebyshev spectral collocation approximation. The combination of very large spatial resolution and long time integration guarantees the quality of the present results. The aim of this work is to provide reference results in order to separate the potential reasons (numerical errors, unsuitable physical or turbulence models) responsible for the long established discrepancy in thermal stratification observed between experimental and numerical estimates using either DNS, LES or RANS simulations.

Noteworthy is the modification of the flow structure by the thermal BCs in the horizontal walls. PAC flow structures are characterized by thin vertical boundary layers along the isothermal walls and two successive separated recirculating flow regions in the upstream part of the horizontal boundary layers. Only a chaotic behavior of the vertical boundary layers has been observed. In the IRC, the flow exhibits two strong recirculating regions along the whole top and bottom walls of the cavity and turbulent vertical boundary layers in their downstream parts. This IRC flow structure as well as the relative $rms$ fluctuations distribution are typical of the flow dynamics observed experimentally.

However both 3D simulations keep resulting in a thermal stratification value equal to one around the cavity mid-height, leading to the conclusion that neither the tridimensional effect nor the experimental distribution on the top/bottom walls improve the quality of numerical prediction concerning the thermal stratification. Moreover, the IRC simulation reveals that the time-averaged results in the vertical mid-depth plane are in good agreement with the previous 2D results. This was also observed by Soria et al. (2004) and Trías et al. (2007) concerning the general features of the flow when performing 2D and 3D DNS with periodicity in the spanwise direction in an adiabatic cavity (PAC) of aspect ratio 4 for a Rayleigh number up to $10^{10}$. This confirms, in agreement with Fusegi and Hyun (1994), that realistic thermal BCs on the top and bottom walls are not key factors for explaining the weak vertical stratification observed in the experimental studies.

The conclusion that can be drawn from these results is that both configurations miss a physical phenomenon that prevents from recovering the experimental data. The influence of the thermal BCs applied on the front and rear walls will be the subject of the Part II article (Sergent et al., 2013).

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References


