Large Eddy Simulation of turbulent thermal convection using a Mixed Scale Diffusivity Model

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Abstract: The Mixed Scale Diffusivity Model, originally developed in the case of the differentially heated cavity, is applied to compute turbulent Rayleigh-Bénard flow in an infinite fluid layer at \(Pr = 0.71\) for a large range of Rayleigh numbers \(6.3 \times 10^5 - 2 \times 10^{11}\). The effect of this SGS modelling, which adjusts locally the SGS diffusivity to the thermal scales of the flow and results in variable \(Pr_{SGS}\) like the dynamic approach, is emphasised by comparison with LES and TRANS literature data. A single scaling regime is found in a range of Rayleigh numbers \(6.3 \times 10^5 - 2 \times 10^{10}\), whose properties include the \(Ra^{0.302}\) scaling law for the Nusselt number and for the thermal boundary layer thickness, in agreement with the experimental correlation of Niemela et al. (2000). The first indication of a transition towards a new regime appears above \(Ra = 10^{11}\).

Keywords: thermal convection; Rayleigh-Bénard; turbulence modelling; large eddy simulation.


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1 Introduction

Rayleigh-Bénard convection occurs in a layer of fluid submitted to a gravity field heated from the bottom and cooled at the top. An unstable vertical density gradient settles across the layer, which provides the buoyancy force and triggers convective motions. This situation is commonly encountered in industrial and environmental applications involving heat exchange. Over the past few decades, great effort has been devoted to the study of convective turbulence in the fully developed condition (Siggia, 1994) in order to determine the Rayleigh number \(Ra\) influence upon the heat transfer estimated through the Nusselt number \(Nu\).
However much controversy always surrounds the physics of turbulent Rayleigh-Bénard convection, especially at high Rayleigh numbers (Chavanne et al., 2001; Niemela and Sreenivasan, 2003), concerning the identification of the different turbulent regimes \((Nu \approx Ra^3)\) and the corresponding flows.

Numerical simulations can supplement experiments as effective tools to clarify the nature of the large scale vortices at large \(Ra\). But the flow is governed by dominant and well-organised large-scale structures. Capturing these structures can only be done by performing time-dependent simulations, such as direct numerical simulation (DNS), large eddy simulation (LES) or the time-dependent statistical approach (Transient Reynolds Averaged Navier Stokes method, T-RANS). Because of the necessity to resolve accurately the thin wall boundary layer and the wall heat transfer, the DNS application is limited to moderate Rayleigh number (for the most recent computations: \(Ra \sim 10^7\) (Kerr, 1996) in an infinite fluid layer and \(Ra \sim 10^{11}\) (Verzicco and Camussi, 2003) but in cylinder higher than wide, i.e. aspect ratio \(AR\) smaller than 1). An alternative way is to resolve in time and space the large coherent eddy structures while the unresolved scales are modelled. In this way, a T-RANS approach using a conventional algebraic stress and flux closure as ‘subscale’ model was applied by Kenjeres and Hanjalic (1999, 2002) to predict the Rayleigh-Bénard flow. Some LES computations were performed (Eidsion, 1985; Wong and Lilly, 1994; Kimmel and Domaradzki, 2000) with the purpose of validate subgrid models. Indeed, computing flows where the temperature and velocity gradients are vertical places severe constraints on the turbulence model. These two turbulence modelling approaches are very appealing insofar as they are potentially able to reach very high Rayleigh numbers corresponding to the \(Nu \approx Ra^{0.38}\) regime experimentally observed by Chavanne et al. (2001) for \(10^4 < Ra < 10^{14}\), like the results obtained by Kenjeres and Hanjalic (2002) up to \(Ra = 10^{15}\).

The aim of this paper is to apply the proposed Mixed Scale Diffusivity Model to the Rayleigh-Bénard convection. This model was validated for the vertically differentially heated cavity (Sargent et al., 2003), for which it improves the prediction of the transitional boundary layer. The comparison exercise will be based on the evaluation of the transitional boundary layer. DNS solutions available (Christie and Domaradzki, 1994; Kerr, 1996) in the literature will be taken as reference solutions.

2 Governing equations

The governing equations for the Large Eddy Simulation of an incompressible fluid flow under Boussinesq assumption are classically derived by applying a convolution filter to the unsteady momentum and energy equations. The resulting set of non-dimensional equations reads:

\[
\frac{\partial \overline{u}_i}{\partial x_i} = 0
\]  

\[(1a)\]

\[
\frac{\partial \overline{u}_i}{\partial x_i} + \frac{\partial \overline{u}_j}{\partial x_j} = -\frac{\partial \overline{p}}{\partial x_i} + \frac{\partial}{\partial x_i} \left( Pr Ra^{1/2} \left( \frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i} \right) \right) - \frac{\partial \tau_{ij}}{\partial x_j} + Pr \frac{\partial \overline{\theta} \delta_{ij}}{\partial x_j},
\]  

\[(1b)\]

\[
\frac{\partial \overline{\theta}}{\partial t} + \frac{\partial \overline{u}_i \overline{\theta}}{\partial x_i} = \frac{\partial}{\partial x_j} \left( Ra^{-1/2} \frac{\partial \overline{\theta}}{\partial x_j} \right) - \frac{\partial h_j}{\partial x_j}.
\]  

\[(1c)\]

These equations were made dimensionless by using the height of the cavity, \(H\), as reference length, the temperature difference between the two vertical isothermal walls of the cavity, \(\Delta T\), and the natural convection characteristic velocity,

\[U_{CV} = \frac{\alpha}{H} Ra^{1/2}.\]

The Rayleigh number is defined as \(Ra = \beta g \Delta T H^3 / \nu \alpha\), and the Prandtl number as \(Pr = \nu / \alpha\), where \(\beta\) is the volumetric thermal expansion coefficient, \(g\) is the acceleration due to the gravity, and \(\nu\) and \(\alpha\) are respectively, the molecular viscosity and diffusivity.

The effects of the subgrid scales removed by the filtering operation on the resolved quantities (\(\overline{\cdot}\)) are accounted for by the subgrid-scale (SGS) stress tensor \(\tau_{ij}\) and heat flux vector \(h_j\):

\[
\tau_{ij} = u_i \overline{u}_j - \overline{u}_i \overline{u}_j,
\]

\[
h_j = \overline{\theta} \overline{u}_j - \overline{\theta} \overline{u}_j.
\]

3 Subgrid-term modelling

3.1 Subgrid stress tensor

All the SGS models studied in the present work belong to the eddy-viscosity family, and therefore assume a linear relationship between the deviatoric part of the subgrid tensor \(\tau\) and the resolved strain-rate tensor \(\overline{\varepsilon}\): \(\tau_{ij} = -2\nu_{SGS} \overline{\varepsilon}_{ij}\), where \(\nu_{SGS}\) is the subgrid viscosity and

\[
\overline{\varepsilon}_{ij} = \frac{1}{2} \left( \frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i} \right).
\]

The subgrid viscosity \(\nu_{SGS}\) is often formulated in terms of the filter width, taken equal to the computational mesh size \((\Delta = (\Delta, \Delta, \Delta)^{1/3})\), and a time scaling quantity, \(T\) which reads as \(\nu_{SGS} = \Delta^2 / T\).
The most common SGS model, the Smagorinsky model (1963) sets the time scaling equal to the reciprocal of the magnitude of the resolved strain rate:

$$\frac{1}{T} = C \left| \bar{S} \right|$$

(4)

where $C$ is the model coefficient and $\left| \bar{S} \right| = \sqrt{2 S_{ij} S_{ij}}$, giving $v_{SGS} = C \alpha^{-1} \left| \bar{S} \right|$ where $C$ is a constant equal to 0.01. This formulation relies on the hypothesis of a local equilibrium of the subgrid scales between the dissipation rate and the shear production of the SGS kinetic energy. Some modifications of the Smagorinsky model have been proposed (Lilly, 1962; Eidson, 1985) in order to take into account the buoyancy production term. However no evidence of real improvement have been shown (Eidson, 1985; Siegel and Domaradzki, 1994). Moreover in the case of a plume simulation (Bastiaans et al., 2000), its use results in a stronger deviation with the reference data than the classical Smagorinsky model. Consequently we have chosen to consider that the subgrid scales are not affected by the buoyancy force.

In order to get proper near-wall behaviour or to simulate flows which are not fully turbulent, the subgrid model should be able to adjust itself to locally inhomogeneous flow. This is usually done by using a Germano-Lilly dynamic procedure (Germano et al., 1991; Lilly, 1992) in which the Smagorinsky constant is computed with respect to the local flow conditions. Wong and Lilly (1994) applied the dynamic model to convective flows and observed improved results over the standard Smagorinsky model. However the dynamic model is less stable than the Smagorinsky model and requires spatial averaging over the local flow homogeneity and bounding limitations.

In this work we chose to adapt the subgrid model to local flow conditions by using the Mixed Scale Viscosity Model, developed by Sagaut (1996). This model stems from a Smagorinsky model (1963) in which the local adaptation is achieved by taking explicitly into account the kinetic energy at the cut-off, $q_\alpha$. The subgrid viscosity is then evaluated as:

$$v_{SGS} = 0.04 \bar{\Lambda}^{-3/2} \left| \bar{S} \right|^{1/2} q_\alpha^{-1/4}$$

(5)

where $q_\alpha$ stands for the kinetic energy at the cut-off, $q_\alpha = (1/2) \bar{\pi}''^2 \bar{\pi}''$. Following Bardina’s similarity hypothesis (Bardina et al., 1980), the velocity field at the cut-off, $\bar{\pi}''$, can be estimated by filtering the resolved velocity field with a test filter coarser than the implicit one, $\bar{\Lambda} > \bar{\Lambda}$, and so $\bar{\pi}'' = \bar{\pi} - \bar{\Lambda}$.

The subgrid-scale dependency of this model ensures that it will adapt to the local state of the flow, and vanish in fully resolved regions of the flow and near the walls.

3.2 Subgrid heat flux vector

Most subgrid models are based on the definition of an eddy diffusivity model and relate the subgrid heat flux components, $h_i$ to the large scale temperature gradient by a SGS diffusivity, $\alpha_{SGS}$, as $h_i = -\alpha_{SGS} (\partial \theta / \partial x_i)$. Usually, $\alpha_{SGS}$ is computed from $v_{SGS}$ assuming a Reynolds analogy $\alpha_{SGS} = v_{SGS} Pr_{SGS}$ with the introduction of a subgrid Prandtl number which is approximated to be a constant. This amounts to consider that the small thermal scales depend solely on the resolved dynamic scales. Therefore it is suspected not to hold in the case of turbulent natural convection, where the flow is produced by buoyant forces and not through a dynamic forcing.

We have thus developed a local subgrid diffusivity model (Sergent et al., 2003), along the lines of the above described Mixed Scale Viscosity Model (Sagaut, 1996). In this approach, the subgrid diffusivity is expressed as a weighted geometrical average of two contributions. The first one comes from the Smagorinsky model using the resolved thermal scales (Smagorinsky, 1963), while the second one comes from the Turbulent Kinetic Energy model (Bardina et al., 1980) based on the subgrid scales. As the functional approach of Smagorinsky assumes that the most important effect of the interaction between the resolved and the subgrid scales is the energy exchange, we focus on the transport equation of the SGS heat flux energy: $\Phi_{SGS} = (1/2) h_i h_j$. The identification of the SGS time scale $T_{SGS}$ leans on the hypothesis of a local equilibrium of the subgrid scales between the dissipation rate and the production of the SGS heat transfer energy:

$$\frac{1}{T_{SGS}} = C'_\phi \frac{\bar{\Lambda}}{\Delta \bar{\theta}} \left| \bar{F} \right|$$

(6)

where

$$\left| \bar{F} \right| = \sqrt{2 T_{SGS} \bar{\pi}''}$$

with $T_{SGS} = \frac{1}{2} \left( \frac{\partial \bar{\theta}}{\partial x_i} + \frac{\partial \bar{\theta}}{\partial x_j} \right) \bar{\pi}''$

(no summation on $i$ and $j$) and $C'_\phi$ is a constant. The isotropic scalar $\left| \bar{F} \right|$ represents the interactions between a temperature gradient and the flow strain.

The combination of the time scaling approach with the scale similarity approach (Bardina et al., 1980) applied to the heat flux energy at the cut-off,

$$\Phi_e = 0.5 \overline{(u \partial \bar{\theta}) (u \partial \bar{\theta})}$$

leads to the following Mixed Scale Diffusivity Model (MSDM):

$$\alpha_{SGS} = C_\alpha \frac{\bar{\Lambda}}{\Delta \bar{\theta}} \left| \bar{F} \right|^{1/2} \left| \Phi_e \right|^{1/4}$$

(7)

with $C_\alpha$, a constant taken equal to 0.5 (Sergent et al., 2003). Using the assumption that the velocity and the temperature
fields are not correlated at the cut-off \((\bar{\vec{u}}\theta'' = \bar{\vec{u}}'' \theta'')\), the heat flux at the cut-off can be evaluated by filtering the resolved temperature and velocity fields using the test filter \(\tilde{\vec{A}}\) like the kinetic energy at the cut-off: \(\Phi_s = q_s (\vec{A} - \tilde{\vec{A}})^2\).

This model has the special feature to be based on its own SGS time scaling, and is independent of the subgrid viscosity and any constant SGS Prandtl number. In order to have a subgrid diffusivity model applicable to any passive or active scalar transport in an incompressible flow, we have assumed the two same hypothesis as those of the Mixed Scale Viscosity model (Sagaut, 1996): (i) a local equilibrium of the subgrid scales between the dissipation and the production rates of the SGS heat transfer energy, and (ii) the Bardina’s similarity hypothesis (Bardina et al., 1980). Moreover the introduction of the heat flux energy at the cut-off in the modelling allows the subgrid diffusivity to vanish at solid walls or when the heat flux field is fully resolved, like the Mixed Scale Viscosity model.

4 Numerical schemes

Time integration of the governing equations (Eqns. 1(b), 1(c)) is performed using a second-order accurate semi-implicit method. The time-stepping scheme combines a second-order backward Euler scheme for the advance in time, an implicit formulation for the diffusion terms and an explicit second-order Adams-Bashforth extrapolation for the nonlinear terms. Incompressibility is imposed by a projection method (Kim and Moin, 1985; Timmermans et al., 1996).

The momentum and scalar balances are discretised using a finite-volume approach on staggered grids. All the terms involved in the balance equations are evaluated with a second-order accurate centred scheme, but a QUICK scheme (Leonard, 1979) is used for the nonlinear terms of the momentum equations. This will be discussed later. The resulting discretised system is solved by an alternating direction implicit method (Peaceman and Rachford, 1955; Douglas, 1957) using trigonometric expansions in the two horizontal homogeneous directions and the Thomas algorithm in the vertical direction. The Poisson equation resulting from the projection method is solved using a direct method.

5 Simulation characteristics

Large eddy simulations were performed in a three-dimensional rectangular domain with periodic boundary conditions in the horizontal directions and no-slip velocity on the isothermal top and bottom plates (\(\theta_t = -0.5\) and \(\theta_b = 0.5\) respectively). The molecular Prandtl number set to 0.71 is assumed for all computations. Because strong vertical shear is concentrated near the horizontal boundaries, we chose a fine vertical resolution near the walls to capture the turbulence production mechanisms, so that no wall model and consequently no additional assumption is needed. Thus the x- direction grid was refined near the walls using a hyperbolic tangent law, while uniform grid distributions were used in the homogeneous y- and z- directions in which periodic boundary conditions were imposed.

Computations were performed using a QUICK scheme for the advective terms in the momentum equations to avoid ‘numerical instabilities’, though the upwind schemes are known to introduce numerical dissipation in the resolved scales. Using an a priori test, it has been shown in Sergent et al. (2003) that this artificial dissipation was able to reproduce correctly the energy transfer between resolved and unresolved scales of the flow, which can be quantified by the time-averaged production of subgrid kinetic energy. Therefore, we chose not to introduce a subgrid viscosity in the momentum equations when using the Mixed Scale Diffusivity Model (MSDM), because the SGS-diffusivity model does not refer to a SGS-viscosity. These simulations were performed on a NEC-SX5 supercomputer with running performances after optimisation of 3.3 GFlops. The 130 × 130 × 130 mesh required 50 CPU hours on a single processor to achieve good statistical convergence.

All the following plots show horizontally averaged vertical profiles unless otherwise noted. A spatially horizontally averaged quantity is denoted by \(< \cdot \cdot \cdot >\), whereas a time fluctuating quantity is denoted by \((\cdot)\).

6 Results

6.1 Validation: \(Ra = 6.3 \times 10^5\) and \(Ra = 2 \times 10^7\)

Before the discussion of the SGS model influence, it is necessary to establish the validity of the code in predicting turbulent Rayleigh-Bénard flows. Few reference data (Christie and Domaradzki, 1994; Kerr, 1996) are available in the literature in an infinite fluid layer (\(Pr = 0.7\)), and concern flows at moderate Rayleigh numbers due to the limitations of the computational resources. Consequently the validation deals with verifying the ability of the SGS model to produce solution close to DNS data even if the Rayleigh numbers are moderate. However the SGS model property to converge towards a DNS-like solution at moderate Rayleigh number is an essential condition to hope to reproduce transitional flow or not fully developed turbulent flow.

The aspect ratio \(AR = L/H\) of the horizontal periodicity to the height of the box is equal to 6. It should be noted that the data sampling procedure to obtain the statistics of the flow was not started until the flow was fully developed. Table 1 summarises the computational details of the spectral DNS carried out by Christie and Domaradzki (1994) and Kerr (1996), and the present LES computations.
Figure 1 presents the comparison of vertical profiles of the temperature variance \(\langle \theta' \rangle\) and vertical velocity fluctuations \(\langle U' \rangle\). Generally speaking the rms values are under-predicted at mid height of the cavity. This can be interpreted as the consequence of the scale filter \((\tau)\) of the LES equations, as already observed by Kimmel and Domaradzki (2000). Indeed, when these authors computed filtered profiles from the DNS data of Christie and Domaradzki (1994), they observed a general small decrease of the statistics quantities. If we consider now the thermal boundary layer thickness \(\delta_\theta\), defined as the distance of the rms peak from the wall, we observe that \(\delta_\theta\) is particularly well estimated. Nevertheless, the maximum of \(\langle \theta' \rangle\) at \(Ra = 2 \times 10^7\) is over-predicted.

Moreover it can be seen in Table 1 that the present LES of the global Nusselt number \(\langle Nu \rangle\) is about 2–9% higher than the DNS reference. But, as all published LES results (Eidson, 1985; Wong and Lilly, 1994; Kimmel and Domaradzki, 2000), over-predict \(\langle Nu \rangle\) by about 25%, we consider these differences weak enough to validate our SGS modelling at low and moderate Rayleigh numbers.

<table>
<thead>
<tr>
<th>Ra</th>
<th>(N_f)</th>
<th>(N_H^2)</th>
<th>(\langle Nu \rangle)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.3 (\times 10^5)</td>
<td>65</td>
<td>128^2</td>
<td>7.2</td>
</tr>
<tr>
<td>Christie and Domaradzki (1994) and Kimmel and Domaradzki (2000)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 (\times 10^7)</td>
<td>64</td>
<td>64^2</td>
<td>7.4</td>
</tr>
<tr>
<td>Present study</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 (\times 10^7)</td>
<td>96</td>
<td>288^2</td>
<td>19.3</td>
</tr>
<tr>
<td>Kerr (1996)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Present study</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(N_f\) and \(N_H^2\) numbers of grid cells in the vertical and horizontal directions.

**Figure 1** Vertical profiles of the mean temperature variance \(\langle \theta' \rangle\) and vertical velocity fluctuations \(\langle U' \rangle\), left: \(Ra = 6.3 \times 10^5\), right: \(Ra = 2 \times 10^7\).

### 6.2 SGS model effects on turbulence quantities

In order to exhibit the behaviour of the proposed SGS approach (Mixed Scale Diffusivity Model) and to compare with literature data, a series of additional computations have been performed with the classical SGS models, described in Section 3, at \(Ra = 10^7\) and \(Ra = 10^9\). The grid sizes are respectively, \(64^3\) and \(96^3\), with an aspect ratio taken equal to 6 in order to limit the influence of the periodic boundary conditions (Grötzbach, 1983; Kerr, 1996).

We compare the influence of two different approaches of the SGS thermal modelling by using first, the Mixed Scale Diffusivity Model (MSDM) without any SGS viscosity model, and secondly the Reynolds analogy (REY) with the Mixed Scale Viscosity Model. The SGS Prandtl number \(P_{SGS}\) is taken to be equal to 0.6 as recommended for passive scalar (Lesieur, 1990), or in the case of thermal plumes simulation (Bastiaans et al., 2000). Additionally, a coarse DNS (CDNS) without any SGS-
model was performed to assess the importance of the models in the simulations. Moreover literature results of computations performed with turbulence modelling are available in this configuration: Eidson (1985) carried out large eddy simulations with a modified Smagorinsky model including a buoyancy term coupled with a Reynolds analogy, Kimmel and Domaradzki (2000) proposed an estimation model applied on the SGS stress tensor as well as on the SGS heat flux vector and finally TRANS computations have been performed using an algebraic low-Reynolds-number model (Kenjeres and Hanjalic, 1999; 2002).

First, it is difficult to distinguish our three computations in the evaluation of the temperature variance (Fig. 2) except for the CDNS profile at \( Ra = 10^9 \), which over-predicts the maximum value of \( \theta' \) in the thermal boundary layer by comparison with MSDM or REY, but under-predicts it in the core of the cavity. It emphasizes the SGS-models influence: \( \alpha_{SGS} \) increases the thermal fluctuations in the bulk but reduces it in the thermal boundary layers.

The estimation model, unlike the Eidson results, predicts lower \( \langle \theta' \rangle \) in the bulk of the cavity as well as at \( Ra = 6.3 \times 10^5 \) as \( Ra = 2 \times 10^7 \). In the same way, TRANS leads to lower thermal fluctuations particularly in the core of the cavity. But the lack of reference data at these two Rayleigh numbers does not allow to evaluate the prediction ability of these SGS and TRANS turbulence approaches. However the MSDM \( \langle \theta' \rangle \) profile at \( Ra = 6.3 \times 10^5 \) (see Fig. 1) is closer to the DNS profile than the profile obtained by the estimation model (Kimmel and Domaradzki, 2000).

Figure 2  Mean temperature variance profiles \( \langle \theta' \rangle \). Left: \( Ra = 10^7 \), right: \( Ra = 10^9 \)

The time-averaged subgrid diffusivity profiles are plotted in Figure 3. The mixed scale approach (MSDM and REY) provides weaker values of the \( \langle \alpha_{SGS}/\alpha \rangle \) than those obtained by Eidson (1985) using the Reynolds analogy associated to the Smagorinsky model. These values are widely influenced by the spatial resolution of the computation, which is four times coarser for the Eidson simulations than in our case. Moreover the Eidson highest values correspond to the boundary layer location, whereas the REY-profile decreases in this area. It means that for the Mixed Scale Model (Eq. 5), the kinetic energy evaluated at the cut-off \( q_c \), which ponders the too diffusive Smagorinsky model, acts in the thermal boundary layer, and particularly where the grid is sufficiently refined by the hyperbolic tangent law. Moreover the fact that the MSDM- and REY-diffusivity values are weak, explain the difficulty to distinguish their respective results even at \( Ra = 10^9 \). Computations at higher Rayleigh number are needed to assess their relative behaviours.

Figure 3  Mean subgrid-scale thermal diffusivity profile \( \langle \alpha_{SGS}/\alpha \rangle \). Left: \( Ra = 6.3 \times 10^5 \), right: \( Ra = 10^7 \)
However we can compare the relative SGS thermal time-scales: \( \frac{1}{\tau_\theta} \) (Eq. 6) for the Mixed Scale Diffusivity model and \( \frac{1}{\tau_\theta} = \frac{1}{T_\theta} \times \frac{1}{Pr_{SGS}} \) for the Reynolds analogy. These time-scales are plotted in Figure 4 as well as the SGS kinetic timescale (\( \frac{1}{\tau_\eta} \), Eq. 4). In the case of the Reynolds analogy, the frequencies are larger in the boundary layers than in the core of the cavity, which emphasises the presence of small scales in the boundary layers. Moreover, as the Reynolds analogy leans on a subgrid Prandtl number \( Pr_{SGS} \) lower than 1, there is only a \( \frac{1}{Pr_{SGS}} \) ratio between the thermal and the kinetic frequencies, corresponding to the assumption that the thermal SGS-scales are smaller than the kinetic SGS-scales. This is coherent with the relative boundary layers thicknesses, but not with the Verzicco and Camussi’s (2003) results for the vertical variation of turbulent length scales averaged on the horizontal plane. Indeed they observe that the turbulent kinetic \( \langle L \rangle \approx \langle U' \rangle^{3/2}/\varepsilon \) and the turbulent thermal length scales \( \langle L_\theta \rangle \approx \langle \theta' \rangle^{2}/N \) (\( \varepsilon \) is the turbulent energy dissipation, and \( N \) the temperature variance dissipation) evolve in a different way depending on the flow dynamics: in the boundary layer \( L > L_\theta \) whereas \( L < L_\theta \) in the core of the cavity. This is in good agreement with the vertical variation of MSDM-thermal and kinetic frequencies for which the thermal frequency is larger than the kinetic one in the boundary layer, while it is the opposite in the core of the cavity. The Mixed Scale Diffusivity Model thus acts like a dynamic approach (Wong and Lilly, 1994), insofar as it results in a variable SGS Prandtl number.

6.3 Heat transfer

In order to demonstrate the ability of our LES to reproduce scaling law and turbulence transition, we have performed over a large range of Rayleigh numbers computations using the Mixed Scale Diffusivity Model, whose characteristics are presented in Table 2. An aspect ratio taken equal to 6 is preconised in Grötzbach (1983) and Kerr (1996) to limit the influence of the periodic boundary conditions. But to maintain a reasonable spatial resolution, we reduce the aspect ratio to 4 for the higher \( Ra \).

In Figure 5 we report the non-dimensional heat transfer as a function of Rayleigh number with some experimental results (Chavanne et al., 2001; Niemela and Sreenivasan, 2003; Fleischer and Goldstein, 2002) for comparison. The LES results show a good agreement with experimental correlations over the \( Ra \) range \( 6.3 \times 10^5 - 2 \times 10^{10} \), and particularly with Niemela’s correlation (Niemela et al., 2000). The best fit to our data is given by \( Nu = 0.131 Ra^{0.302} \). However our \( Nu \) values under-estimate the experimental data, but this is due to the aspect ratio dependence of the Nusselt number (Niemela and Sreenivasan, 2003). Besides we do not obtain the \( -2/7 \) power law of the hard turbulent regime described experimentally by Chavanne et al. (2001) or numerically by the DNS of Kerr (1996).

### Table 2: Simulation characteristics

<table>
<thead>
<tr>
<th>( Ra )</th>
<th>( N_v \times N_{H}^2 )</th>
<th>( AR )</th>
<th>( dt )</th>
<th>( \eta )</th>
<th>( \Delta \eta_{MAX}/\eta )</th>
<th>( \Delta H/\eta )</th>
<th>( \Delta\eta/\Delta H )</th>
<th>( \langle Nu \rangle )</th>
</tr>
</thead>
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<tr>
<td>( 2 \times 10^6 )</td>
<td>52 × 96²</td>
<td>6</td>
<td>0.02</td>
<td>0.0393</td>
<td>1.0</td>
<td>1.6</td>
<td>21</td>
<td>10.3</td>
</tr>
<tr>
<td>( 2 \times 10^7 )</td>
<td>64 × 96²</td>
<td>6</td>
<td>0.01</td>
<td>0.0185</td>
<td>1.9</td>
<td>3.3</td>
<td>38</td>
<td>21.0</td>
</tr>
<tr>
<td>( 2 \times 10^8 )</td>
<td>80 × 96²</td>
<td>6</td>
<td>0.01</td>
<td>0.0087</td>
<td>3.6</td>
<td>7.0</td>
<td>76</td>
<td>42.0</td>
</tr>
<tr>
<td>( 2 \times 10^9 )</td>
<td>96 × 128²</td>
<td>4</td>
<td>0.006</td>
<td>0.0042</td>
<td>6.5</td>
<td>7.4</td>
<td>75</td>
<td>81.5</td>
</tr>
<tr>
<td>( 2 \times 10^{10} )</td>
<td>112 × 128²</td>
<td>4</td>
<td>0.006</td>
<td>0.0020</td>
<td>8.8</td>
<td>15.8</td>
<td>157</td>
<td>169.6</td>
</tr>
<tr>
<td>( 2 \times 10^{11} )</td>
<td>128 × 128²</td>
<td>4</td>
<td>0.004</td>
<td>0.0009</td>
<td>10.2</td>
<td>34.3</td>
<td>299</td>
<td>381.3</td>
</tr>
</tbody>
</table>

\( N_v \) and \( N_{H} \): numbers of grid cells in the vertical and horizontal directions.

\( dt \): time step.

\( \eta \): Kolmogorov scale (\( \eta = H (Pr^2/RaNu)^{1/4} \) (Grötzbach, 1983).

\( \Delta \eta_{MAX} \): maximum of the vertical spacing.

\( \Delta H \) and \( \Delta \eta \): horizontal mesh size and vertical mesh size at wall.

\( Nu \): Nusselt number at wall.
The thermal boundary layer thickness (Fig. 6) has a similar power law $Ra$ dependence than the Nusselt number. Assuming the boundary layer thickness $\delta$ to be the distance from wall of the peak of the rms temperature vertical profile, we do observe the $Ra^{-0.302}$ scaling law of the thermal boundary layer thickness. Moreover the velocity boundary layer thickness $\delta_v$ (estimated as the distance from wall of the peak of the vertical profile of the rms horizontal velocity) displays a $Ra^{-(1/7)}$ scaling law over a large range of Rayleigh numbers $2 \times 10^6 - 2 \times 10^9$, which is in good agreement with DNS (Kerr, 1996) and TRANS results (Kenjeres and Hanjalic, 2002). However, above $Ra = 10^{10}$, our simulations indicate a change in $\delta_v$ slope, corresponding to an increase in the slope of $Nu(Ra)$ correlation in Figure 5. This could be interpreted as well as an indication of a possible new regime, as done by Kenjeres and Hanjalic (2002), or as an indication of an insufficient spatial resolution.

**Figure 5** Comparison of the computed global Nusselt number with experimental correlations

In order to underline the influence of our SGS model, we compare the Nusselt numbers obtained in a large aspect ratio fluid layer by the DNS performed by Kerr (1996) and by different turbulence modelling (TRANS or LES) with our results (Fig. 7). First we observe a good agreement with the DNS correlation especially for the lowest $Ra$, whereas the literature data using a turbulence modelling over-predict the DNS values. The Eidson’s LES results (Eidson, 1985) become anomalous at $Ra = 2.5 \times 10^6$, which is attributed by Eidson to an insufficient spatial resolution, because the first vertical grid point was outside the conductive sublayer. Concerning the Kimmel and Domaradzki’s (2000) results, the over-prediction of the Nusselt number directly originates from the SGS model through the evaluation of the SGS heat flux. For Rayleigh numbers higher than $10^8$, the only available numerical results are the TRANS simulations performed by Kenjeres and Hanjalic (2002). They observe an enhancement in the Nusselt number for $Ra > 2 \times 10^{13}$ compared with the $Nu(Ra)$ trend at lower Rayleigh together with a change in the slope of the velocity boundary layer thickness. Kenjeres and Hanjalic interpret these two observations as an indication of a possible new regime. This critical Rayleigh number is two orders of magnitude higher than our observation, whereas Kenjeres and Hanjalic’s computations have been performed in a $8-AR$-box, that is larger than our computational domains (see Table 2). Yet Castaing et al. (1989) noticed that the $Nu$ power law behaviour starts from lower $Ra$ values with increasing $AR$. Consequently, we conclude that only the spatial discretisation or the turbulence modelling can explain the discrepancies between our results and those of Kenjeres and Hanjalic. An insufficient vertical description of the conductive sublayer results in an asymptotic behaviour of the Nusselt number when the Rayleigh number increases (Eidson, 1985). Concerning our computations, the Nusselt number at wall seems not to be affected. But we observe an asymptotic behaviour of the thermal boundary layer thickness. The ratio horizontal mesh size by the first cell size ($\Delta_H/\Delta_w$) which is given in the Table 2, leads to
wonder if the tangential thermal dissipation is not still important for the conductive sublayer at high Rayleigh number, even if the viscous sublayer does not seem concerned by this cell distortion. Further computations are needed to determine the required mesh resolution to obtain reliable results.

7 Conclusions

The Mixed Scale Diffusivity Model, originally developed in the case of the differentially heated cavity, was applied to compute turbulent Rayleigh-Bénard flow in an infinite fluid layer at $Pr = 0.71$ for a large range of Rayleigh numbers ($6.3 \times 10^5 - 2 \times 10^{11}$). Before the discussion of the SGS model influence, we establish the validity of the code in predicting turbulent Rayleigh-Bénard flows by comparison of vertical profiles of the rms of the temperature and vertical velocity fluctuations with DNS data.

The SGS modelling effect is then emphasised by comparison with LES and TRANS literature data. As the Mixed Scale Diffusivity Model is independent of the subgrid viscosity so that the Reynolds analogy is not needed; it adjusts locally the SGS diffusivity to the thermal scales of the flow, and consequently is not very dissipative, which leads to a lower estimation of Nusselt numbers than other turbulence modelling of the literature. Moreover a variable $Pr_{SGS}$ is obtained, as with a dynamic approach.

A single scaling regime is found in the range of Rayleigh numbers $6.3 \times 10^5 - 2 \times 10^{11}$. Generally speaking, the present correlation $Nu(Ra)$ is in good agreement with previous DNS and experiments. The properties of the observed regime include the $Ra^{0.302}$ scaling law for the Nusselt number and for the thermal boundary layer thickness, in agreement with the experimental correlation of Niemela et al. (2000). Above $Ra = 10^{10} - 10^{11}$ the first indication of a transition towards a new regime appears with the increase of the heat transfer and a change in the $\delta_b$ slope. But the fact that this critical Rayleigh number is two orders of magnitude below the Kenjeres and Hanjalic observation in an 8-AR box leads us to wonder about the reason for this transition. Is it a turbulence modelling or an insufficient spatial resolution effect? In order to answer to this question, further computations where special attention is paid to the spatial discretisation, are underway. The results will be reported elsewhere.

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References


