Numerical study of laminar starting forced plumes at high Schmidt number

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Outline

1. Introduction
2. Configuration
3. Physical modeling
4. Numerical methods
5. Results
1. Introduction
Plumes

Turbulent plumes

Laminar plumes
Steady plume

Starting plume

\[ w_h = k \ w_c , \ k < 1 \]

Thermal plumes: [Shlien, 1976], [Moses et al., 1993], [Kaminski and Jaupart, 2003], compositional forced plumes: [Rogers and Morris, 2009]
Present study

Objective

- Develop a numerical tool to simulate laminar starting plumes at high Schmidt number in a confined space

Method

- Use a DNS numerical code with suitable modifications
- Validate the numerical results with experimental data
2. Configuration
Experimental vs. Numerical

- $T=22\, ^\circ\text{C}, P = 1\, \text{atm}$
- Ambient and injected fluids: glycerol-water solutions

[Rogers and Morris, 2009]
- 3D parallelopipe

[Num]
- 2D axisymmetric
Physical parameters of 13 cases

<table>
<thead>
<tr>
<th>Case</th>
<th>( \frac{\rho_a - \rho_i}{\rho_a} )</th>
<th>( \frac{\nu_i}{\nu_a} )</th>
<th>( Gr )</th>
<th>( Sc_a )</th>
<th>( Re_i )</th>
<th>( Ri_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1,1 - D1,5</td>
<td>0.0026</td>
<td>0.92</td>
<td>( 1.3 \times 10^7 )</td>
<td>64128</td>
<td>0.99-11.96</td>
<td>1.75-0.01</td>
</tr>
<tr>
<td>D2</td>
<td>0.0038</td>
<td>0.88</td>
<td>( 1.7 \times 10^7 )</td>
<td>70976</td>
<td>7.83</td>
<td>0.04</td>
</tr>
<tr>
<td>D4,1 - D4,5</td>
<td>0.0094</td>
<td>0.63</td>
<td>( 2.8 \times 10^6 )</td>
<td>592574</td>
<td>0.37-4.48</td>
<td>6.42-0.04</td>
</tr>
<tr>
<td>D5,1 - D5,2</td>
<td>0.0021</td>
<td>0.90</td>
<td>( 8.2 \times 10^5 )</td>
<td>597078</td>
<td>1.04-1.56</td>
<td>0.09-0.04</td>
</tr>
</tbody>
</table>

- Injection Reynolds number: \( Re_i = \frac{\bar{w}_i d_i}{\nu_i} \)
- Injection Richardson number: \( Ri_i = \frac{1}{2} g \frac{\rho_a - \rho_i}{\rho_i} \frac{d_i}{\bar{w}_i^2} \)
- Grashof number: \( Gr = g \frac{\rho_a - \rho_i}{\rho_a} \frac{H^3}{\nu_a^2} \)
- Schmidt number: \( Sc_a = \frac{\nu_a}{D_a} \)
3. Physical modeling
Governing equations for a binary mixture

\[ \nabla \cdot \mathbf{u} = 0 \]
\[ \frac{\partial \rho Y_1}{\partial t} + \nabla \cdot (\rho Y_1 \mathbf{u}) = \frac{1}{ReSc_a} \nabla \cdot (\rho D \nabla Y_1) \]
\[ \frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) = -\nabla p + \frac{1}{Re} \nabla \cdot (\rho \nu (\nabla \mathbf{u} + \nabla^T \mathbf{u})) + \frac{1}{Fr} \rho z \]
\[ \rho = \rho_1 Y_1 + \rho_2 (1 - Y_1) \]

+ Laws for the variation of \( D \) and \( \nu \) with \( Y_1 \)

- \( Y_1 \): Mass fraction of glycerol, \( \rho \): Mixture density
- \( \mathbf{u} \): Mass-averaged velocity
- \( \rho_1, \rho_2 \): Density of pure glycerol and water
- \( Re = \frac{\bar{w}_i H}{\nu_a} \), \( Fr = \frac{\bar{w}_i^2}{gH} \)
Boundary conditions

\[
\begin{align*}
\frac{\partial u}{\partial z} &= 0 & \frac{\partial w}{\partial z} &= 0 & \frac{\partial Y_1}{\partial z} &= 0 \\
\frac{\partial u}{\partial r} &= 0 \\
\frac{\partial w}{\partial r} &= 0 \\
\frac{\partial Y_1}{\partial r} &= 0
\end{align*}
\]

- $u = 0$
- $w = 0$
- $\frac{\partial Y_1}{\partial r} = 0$

$Y_1 = Y_{1i}$  $u = 0$  $w = \overline{w_i} \left( -\frac{8r^2}{d_i^2} + 2 \right)$
4. Numerical methods
Numerical methods

- **Time discretization**: Semi-implicit, 2nd order BDF2 + Adams-Bashforth, projection method for velocity-pressure coupling.

- **Spatial discretization**: finite volume, staggered grid, 2nd order central differences for diffusive terms, 3rd order QUICK for convective terms.

- Grid: 514x2498 regular, time step: $5 \times 10^{-3}$ seconds.
5. Results
General plume shape

[Rogers and Morris, 2009]

- \( h = 19.3 \text{cm} \)
- \( l_h = 1.6 \text{cm} \)
- \( d_h = 1.9 \text{cm} \)
- \( d_c = 0.60 \text{cm} \)
- \( RF_h = 1.19 \)

[Num]

- \( h = 19.23 \text{cm} \)
- \( l_h = 1.72 \text{cm} \)
- \( d_h = 1.97 \text{cm} \)
- \( d_c = 0.66 \text{cm} \)
- \( RF_h = 1.15 \)

(*) Head aspect ratio

\[ RF_h = \frac{d_h}{l_h} \]
Time evolution of the plume height

\[ \omega_h \]
Comparison of the ascent velocity

![Image](image.png)

<table>
<thead>
<tr>
<th>Case</th>
<th>$Re_i$</th>
<th>$w_h$</th>
<th>$w_h^e$</th>
<th>$\frac{w_h}{w_h^e}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>D4,1</td>
<td>0.370</td>
<td>0.472</td>
<td>0.47</td>
<td>1.00</td>
</tr>
<tr>
<td>D4,2</td>
<td>0.751</td>
<td>0.636</td>
<td>0.64</td>
<td>0.99</td>
</tr>
<tr>
<td>D4,3</td>
<td>1.491</td>
<td>0.849</td>
<td>0.86</td>
<td>0.99</td>
</tr>
<tr>
<td>D4,4</td>
<td>2.992</td>
<td>1.095</td>
<td>1.10</td>
<td>1.00</td>
</tr>
<tr>
<td>D4,5</td>
<td>4.483</td>
<td>1.302</td>
<td>1.34</td>
<td>0.97</td>
</tr>
</tbody>
</table>

[Num]

[Rogers and Morris, 2009]
Correlation for the ascent velocity

\[ Ri_h = 3.2 Re_i^{-0.91} \]

\[ Ri_h = 3.7 Re_i^{*-0.78} \]

Injection Reynolds
\[ Re_i = \frac{d_i w_i}{\nu_i} \]

Head Richardson
\[ Ri_h = \frac{g(\rho_a - \rho_i)d_i}{\rho_a w_h^2} \]

Modified injection Reynolds
\[ Re_i^* = \frac{d_i w_i}{\nu_a} \]
Time evolution of confined heads

\[ RF_h = 1.15 \]

\[ RF_h = 1.08 \]

(*) Results taken at the same plume height (case D5)
Time evolution of dispersed heads

(*) Results taken at the same plume height (case D2)

$R F_h = 1.19$

$1.6 \text{ cm}$

$2.1 \text{ cm}$

$1.9 \text{ cm}$

$R F_h = 0.74$

[Num]

[Rogers and Morris, 2009]
Overview of head shape

[Rogers and Morris, 2009]

\[ Ri_h > 1 \]

Confined heads

\[ Ri_h < 1 \]

Dispersed heads

<table>
<thead>
<tr>
<th>Case</th>
<th>Type of head</th>
<th>( Ri_h )</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1,1</td>
<td>confined</td>
<td>3.512</td>
</tr>
<tr>
<td>D1,2</td>
<td>confined</td>
<td>2.181</td>
</tr>
<tr>
<td>D1,3</td>
<td>dispersed</td>
<td>1.116</td>
</tr>
<tr>
<td>D1,4</td>
<td>dispersed</td>
<td>0.786</td>
</tr>
<tr>
<td>D1,5</td>
<td>dispersed</td>
<td>0.597</td>
</tr>
<tr>
<td>D5,1</td>
<td>confined</td>
<td>4.235</td>
</tr>
<tr>
<td>D5,2</td>
<td>confined</td>
<td>3.149</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case</th>
<th>Type of head</th>
<th>( Ri_h )</th>
</tr>
</thead>
<tbody>
<tr>
<td>D4,1</td>
<td>confined</td>
<td>12.44</td>
</tr>
<tr>
<td>D4,2</td>
<td>confined</td>
<td>6.854</td>
</tr>
<tr>
<td>D4,3</td>
<td>confined</td>
<td>3.846</td>
</tr>
<tr>
<td>D4,4</td>
<td>confined</td>
<td>2.312</td>
</tr>
<tr>
<td>D4,5</td>
<td>confined</td>
<td>1.635</td>
</tr>
<tr>
<td>D2</td>
<td>dispersed</td>
<td>0.809</td>
</tr>
</tbody>
</table>
Conclusions

- Good numerical model for laminar starting plumes at high Schmidt number
- Constant ascent velocity, within 2 % difference with [Rogers and Morris 2009]
- Lower plume height, probably due to time shift
- A modified correlation for that of [RM09]
- Consistent types of head with [RM09]
- Good agreement of head size for confined heads, poorer agreement for dispersed heads, due to asymmetry
References


Wall effects

\[ \text{Ri}_h = 3.2 \text{Re}_{i}^{* -0.91} \]

\[ \text{Ri}_h = 3.2 \text{Re}_{i}^{* -1} \]

\[ w_h = (0.63 \pm 0.02) \left( \frac{g(\rho_a - \rho_i)Q}{\rho_a \nu_a} \right)^{1/2} \]

Cavity diameter

\[
\begin{array}{cc}
\text{Cavity diameter} & \frac{w_h}{w_f} \\
151 \text{ mm} & 0.84 \\
201 \text{ mm} & 0.87 \\
268 \text{ mm} & 0.90 \\
402 \text{ mm} & 0.93 \\
\end{array}
\]
Head aspect ratio

- \[ RF_h = \frac{d_h}{l_h} \]

- [Num] Always less than 1.24

- [Rogers and Morris, 2009] \( RF_h = 1.24 \) for confined heads

- [Num] Increases with increasing flow rate

- [Num] Decreases with plume height

- [Num] Decreases faster for higher flow rates and dispersed heads